18.02 Recitation
Problems
28 September 2011

The problems with a $(*)$ be the number ask you to consult the topographic map on the back of the sheet. Let $h(x, y)$ denote the the altitude at a point with coordinates (x, y), where the origin is in Ingleby and a unit in the x or y direction is one foot.

1*. How many peaks can you see along the ridge of Sawmill Mountain? How can you see the maximum of a function by looking at its level curves?

2. Describe the level curves of the function $g(x, y) = 4x^2 + y^2$.

3*. You are in downtown Ingleby and want to reach point A. You hike in a straight line (as the crow flies) at a rate of 180 feet per minute (about 2 MPH). Let $e(t)$ denote your elevation at time t, expressed in minutes.

   (a) Sketch a graph of $e(t)$. Where is your altitude maximized? Where is your climb steepest?

   (b) What is $e(t)$, in terms of the function $h(x, y)$?

4. Let $f(x, y) = (x + y)^2 + xy^2 + 2$. Compute the partial derivates $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

5*. Suppose you stand at the point A on the map. What is the meaning of the partial derivative $\frac{\partial h}{\partial x}$? $\frac{\partial h}{\partial y}$? How could you estimate the values of these derivatives?

6. Let f be the function from problem 4. Approximate $f(2+\Delta x, 1+\Delta y)$ using the partial derivatives you computed.

7. Find the tangent plane to the graph of the function f at $(2, 1, 13)$.

8*. Suppose that you want to run a railroad line roughly following the creek to Ingleby. How would you route it? The path should be as short as possible, but is limited to a 2% grade (i.e. slope at most 0.02). What does this have to do with the level curves?

9*. You are at point A and want to climb Paddy Mountain. In what direction is the steepest ascent? Suppose you attack at an angle θ, as illustrated. Estimate the number of feet you will climb per horizontal foot. For what value of θ is this largest? Smallest?

   Can you interpret your answer as a directional derivative of h?