18.02 Recitation
Problems
17 October 2011

1. Consider the function $f(x,y,z) = x + y^2 + z^3$ defined on the sphere $g(x,y,z) = x^2 + y^2 + z^2 = 1$. Find $\left(\frac{\partial f}{\partial x}\right)_y$.

2. (2J-8) Take (r,θ) as polar coordinates. Let $w = \sqrt{r^2 - x^2}$.
   - Compute $\left(\frac{\partial w}{\partial r}\right)_\theta$ by writing w in terms of r and θ.
   - Compute it using the method of differentials.
   - Why does this answer make sense geometrically?

3. Consider a rectangular box with sides x, y, z. Fix the surface area $2xy + 2xz + 2yz$. Let $V(x,y,z) = xyz$ be the volume. Compute $\left(\frac{\partial V}{\partial x}\right)_y \left(\frac{\partial V}{\partial y}\right)_x$.

4. (2J-10) Suppose $u(x,y)$ and $v(x,y)$ are functions of two variables.

   Use differentials to prove the Jacobian rule

$$\left(\frac{\partial u}{\partial x}\right)_v = \left(\frac{\partial u}{\partial x}\right)_y - \left(\frac{\partial u}{\partial v}\right)_x \left(\frac{\partial v}{\partial x}\right)_y.$$

5. Suppose that h is the altitude function. What is the meaning of

$$\iint_R h(x,y)\,dx\,dy$$

   Why can this be computed via evaluating the inner and then outer integral in

$$\int_{y=c}^{d} \int_{x=a}^{b} h(x,y)\,dx\,dy$$

6. Compute the integral of the function $f(x,y) = 2x + y + 1$ over the square $-2 \leq x \leq 2$, $-2 \leq y \leq 2$.

7. What about the same function, integrated over the region between the above square and the square $-1 \leq x \leq 1$.

8. Set up the double integral to compute the integral of $f(x,y) = x \sin y$ over the semicircle of radius 1 centered at $(0,0)$.