This review sheet probably doesn't cover everything you need to know for the exam. Make sure you read through the lecture notes and know how to do all the problems on the practice exams! But this list is one place to start...

- Understand cylindrical and spherical coordinates and be able to convert points and functions between them. Certainly know (or better yet, be able to quickly derive) the formulas

Cylindrical	$x = r\cos\theta,\ y = r\sin\theta,\ z = z$
Spherical	$r = \rho\sin\phi,\ \theta = \theta,\ z = \rho\cos\phi$
Spherical	$x = \rho\cos\theta\sin\phi,\ y = \rho\sin\theta\sin\phi,\ z = \rho\cos\phi$

  You should be able to make the conversions in the other direction as well.

- (5A-1, 5A-2, 5B-1) Be able to set up bounds for triple integrals in all three coordinate systems! Don't forget the volume elements: $dx\,dy\,dz$, $r\,dr\,d\theta\,dz$, and $\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$. The only way to get ready for this is to just do a few problems setting up bounds in the three coordinate systems we've discussed. At the very least, here are a few important ones and others to think about.

Rectangular	$\int_{x=a_1}^{a_2}\int_{y=b_1}^{b_2}\int_{z=c_1}^{c_2} dz\,dy\,dx$	Rectangular box
	$\int_{x=a_1}^{a_2}\int_{y=b_1}^{b_2}\int_{z=0}^{f(x,y)} dz\,dy\,dx$	Region under graph of $f(x,y)$, above the rectangle given in the bounds on x and y.
	...	Area under a plane, area between graphs of two functions, cylinder, sphere
Cylindrical	$\int_{z=c}^{d}\int_{\theta=0}^{2\pi}\int_{r=0}^{a} r\,dr\,d\theta\,dz$	Cylinder of radius a, lying between $z=c$ and $z=d$
	$\int_{\theta=0}^{2\pi}\int_{r=0}^{a}\int_{z=0}^{f(r,\theta)} r\,dz\,dr\,d\theta$	Region under graph of $f(r,\theta)$
		Half cylinder, cone with vertex at origin, sphere, ...
Spherical	$\int_{\phi=0}^{\pi}\int_{\theta=0}^{2\pi}\int_{\rho=0}^{a} \rho^2\sin\phi\,dr\,d\theta\,d\phi$	Sphere of radius a
		Cone with vertex at origin, sphere passing through the origin, cylinder, some segment of a sphere, ...

- (5A-3, 5A-6, 5B-3) Triple integrals have a lot of the same uses that double integrals did. We can find average values of functions, volume, mass, center of mass, moment of inertia:

Property	Integral
Volume	$\iiint_R 1\,dV$
Average value of function	(1/vol) $\iiint_R f(x,y,z)\,dV$
Mass	$\iiint_R \delta(x,y,z)\,dV$
Center of mass: x coordinate	$1/M\ \iiint_R x\delta\,dV$
	(similar for other coordinates)
Moment of inertia about x-axis	$\iiint_R \delta(y^2+z^2)\,dV$
	(similar for other axes)

  When you get one of these, you need to choose the coordinate system in which to compute the integral, express the appropriate integrand in that coordinate system, and plug in the right volume element.

- (5C-3) One application new this unit was gravitation. The strength of the vertical component of the gravitational field from a mass on a particle at $(0,0,0)$ is given by

$$\vec{F} = \iiint_V \frac{G\delta}{\rho^3}\vec{\langle x,y,z\rangle}\cdot\vec{k}\,dV = G\iiint_V \frac{\cos\phi}{\rho^2}\delta\,dV.$$

Look in the lecture notes or supplemental notes G to read about gravitation (the bit in notes G is just a page plus some examples). Of particular note is Newton's theorem, which says that the gravititation attraction from a spherical planet of uniform density is the same as the attraction from a point mass at the center of the sphere. Look at lecture 26.

- (6A-1) We'll also look at vector fields in space. To specify a field, you need to give its $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ parts: $\vec{F} = P\hat{\imath} + Q\hat{\jmath} + R\hat{k}$, where each of these depends on x, y, z. Be able to visualize simple vector fields in space.

- (6B-4, 6B-6) We also define flux, analogously to the two-dimensional version. The integral

$$\text{flux} = \iint_S \vec{F} \cdot \hat{n}\, dS = \iint_S \vec{F} \cdot d\vec{S}$$

measures the amount of flow of $\vec{F}$ across the surface S. Here $\vec{n}$ is a unit vector normal to the surface S (which will be different at different points), and dS is the area element. Note that $\hat{n}\, dS$ is sometimes written all together as $d\vec{S}$. To actually compute flux, you need to find $\hat{n}$ and dS for the appropriate surface. The most important surfaces to keep in mind are these:

Surface	$\hat{n}$	dS
Horizontal plane	$\hat{k}$	$dx\, dy$
Cylinder of radius a	$\frac{1}{a}\langle x, y, 0\rangle$	$a\, d\theta\, dz$
Sphere of radius a	$\frac{1}{a}\langle x, y, z\rangle$	$a^2 \sin\phi\, d\phi\, d\theta$
Graph of $f(x, y)$	$d\vec{S} = \langle -f_x, -f_y, 1\rangle$	

The integral for flux is a double integral, and the bounds of integration will involve two of the coordinates from your coordinate system. For example, if you want flux across a cylinder, you'll integrate for θ from 0 to 2π and z from a to b, while holding r fixed at the radius of the cylinder.

If your region is bounded by a couple different surfaces, you need to break the integral for flux into a few parts and add up the totals for each. To do the computation, take your field $\vec{F}$ (most likely expressed in terms of x, y, and z), and compute the dot product $\vec{F} \cdot \hat{n}$ (again depending on x, y, z). Then you need to write the resulting function in terms of the variables of integration (say, z and θ) and compute the double integral with the appropriate dS.

If the surface isn't on the list above, there's still hope. You need to parametrize it, so the surface is given by $(x(u, v),\ y(u, v))$. Set $\vec{r} = \vec{r}(u, v) = \langle x(u, v), y(u, v)\rangle$ and one checks that $d\vec{S} = \frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v}\, du\, dv$.

- (6C-5, 6C-6) There's a 3-dimensional version of the normal form of Green's theorem, the divergence theorem. It states that

$$\iint_S \vec{F} \cdot \hat{n}\, dS = \iiint_R \operatorname{div} F\, dV$$

Here S is a closed surface and V is the region bounded by the surface. Note that $\vec{F}$ and its divergence should both be continuous here. The divergence of $\vec{F} = P(x, y, z)\hat{\imath} + Q(x, y, z)\hat{\jmath} + R(x, y, z)\hat{k}$ is defined to be the function $\operatorname{div} F = P_x + Q_y + R_z$. The divergence theorem gives lets us compute the flux by doing a triple integral instead (or vice versa). Be sure to practice a couple problems where you compute the same integral both ways!

If you know something about electricity and magnetism, think of this as Gauss's law. Up to some constants, $\vec{F}$ is the electric field, and the divergence of $\vec{F}$ is the charge density.

Watch out for a shorthand here: setting $\nabla = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle$, it's common to write $\operatorname{div} \vec{F} = \nabla \cdot \vec{F}$.