

18.02 Recitation  
Problems  
05 December 2011

1. For what  $a, b, c$  will the field  $\langle ay^2z, yz(bx + z), y^2(x + cz) - 3z^2 \rangle$  be conservative?
2. For what  $a, b, c$  will the differential form  $ay^2z dx + yz(bx + z) dy + (y^2(x + cz) - 3z^2) dz$  be exact?
3. Is the field  $\vec{F} = \langle 2xyz, x^2z + z, x^2y + y + 1 \rangle$  conservative?
4. If your answer to the previous question was “yes”, find a function  $f$  with  $\nabla f = \vec{F}$ . (If your answer was “no”, better check your work). Try to use both the line integral method and the partial derivative method.
5. Let  $\vec{r}(t) = \langle t, t^2, t^3 \rangle$  be the twisted cubic from lecture. Compute  $\int_C \vec{F} \cdot d\vec{r}$  for  $\vec{F} = \langle x, y, z \rangle$ , for  $0 \leq t \leq 1$  directly.
6. Compute  $\int_C \vec{F} \cdot d\vec{r}$ , where  $\vec{r}$  is again the twisted cubic, and  $\vec{F}$  is the field from number 3. Use the fundamental theorem of calculus for line integrals.
7. Parametrize the following paths in three dimensions:
  - (a) A straight line from  $(1, 2, 3)$  to  $(4, 5, 7)$ .
  - (b) A helix of radius  $a$ , rotating with angular velocity 1, moving upwards at a rate of 2.
  - (c) A circle in the plane  $x = -1$ , of radius 4.
  - (d) A line of longitude on the sphere of radius 1 centered at the origin, passing through  $(\sqrt{2}/2, \sqrt{2}/2, 0)$ .