18.02 Recitation
Problems
07 December 2011

1. Describe orientations on the following surfaces:

   (a) A rectangle with vertices at $(0, 0, 0)$, $(1, 1, 0)$, $(0, 0, 1)$, $(1, 1, 1)$.

   (b) A sphere of radius 2, with the region that has $z \geq 1$ removed.

   (c) The part of the paraboloid $z = 1 - x^2 - y^2$ that lies above the xy-plane.

   (d) A sphere with four circular holes cut out.

2. (6F3) Verify Stokes when S is the rectangle from part a) and $\vec{F} = yz\hat{\imath} + xz\hat{\jmath} + xy\hat{k}$.

3. Verify Stokes when S is the region from b) of the first problem, C its boundary, and $F = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$.

4. Consider a cylinder of radius 1, bounded below by the plane $z = 0$ and above by $z = 2 + \cos 2\theta$. Let $\vec{F} = \langle x, y, 0 \rangle$. Use Stokes' theorem to compute the integral $\oint \vec{F} \cdot d\vec{r}$ over the top edge.

5. Let S be a cylinder of height 2 and radius 1 centered at the origin (not including either of the ends!). The region has two boundary components. Find an orientation compatible with both of them, and verify Stokes' theorem for the region, with the field $\vec{F} = \langle yz, -xz, 0 \rangle$.

6. Let S be a Möbius strip. How many boundary curves does it have? How should S be oriented? What about a strip twisted k times?

7. What does Stokes' theorem tell us in the case that $\nabla \times \vec{F} = 0$? Does this make sense?

8. Can you come up with an example of a closed surface (i.e. there are no boundary curves) which can't be oriented?