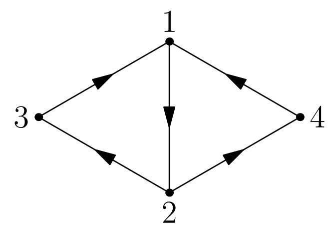
Math 18.06, r3/5
Problems #7
April 11, 2013

1. Let G be the following graph with six nodes.



Write down the incidence matrix A for G. What is the nullspace of A? What is the left nullspace of A, and what does this tell you about the graph?

2. Find an orthonormal basis for the plane in $\mathbb{R}^3$ defined by $x + y + z = 0$. Use your basis to find the matrix for projection onto this plane.

3. Use the cofactor matrix to find the inverse of

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix}$$

4. Find the best fit line through the three points $(0, 0)$, $(1, 0)$, $(1, 1)$.

5. (a) Consider a parallelogram with vertices at $(0, 0)$, $(a, 0)$, (c, d), and $(a + c, d)$. What is its area?
   (b) Let A be a 3×3 matrix with determinant 5. What is the determinant of $-2A^T$?
   (c) Suppose that A is symmetric and P is the projection onto the nullspace of A. What is AP? PA?

6. Consider the following very small-dimensional instance of least squares fit. Let A be the 2×1 matrix $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$, and $\mathbf{b}$ be the vector $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$.

   Find the least-squares solution $A\hat{\mathbf{x}} = \mathbf{p}$ to $A\mathbf{x} = \mathbf{b}$. Compute the vectors $\mathbf{p}$ and $\mathbf{e} = \mathbf{b} - \mathbf{p}$. What's the relation between $\mathbf{e}$ and A?

7. Let

$$A = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

   What is $\det A$? What is the $(1, 2)$-entry of A^{-1}?

8. Suppose that A is an $m \times n$ matrix whose rows are linearly independent. Compute the dimension of the nullspace of $A^T A$.