

1. (a) An example of a matrix that isn't diagonalizable is: _____.
 - (b) If A is symmetric, then it can be diagonalized as _____.
Is every matrix that can be diagonalized in this way symmetric?
 - (c) We had another decomposition for symmetric matrices, $A = LDL^T$, with D diagonal. Is this the same thing as the diagonalization of A ?
 - (d) When is a diagonal matrix positive-definite? Explain why $\mathbf{x}^T D \mathbf{x}$ could be negative if D has a negative entry.
2. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$. Is A diagonalizable? If so, diagonalize it using an orthogonal eigenbasis.
 3. Prove that if A is a skew-symmetric real matrix ($A^T = -A$), then all the eigenvalues of A are purely imaginary. Here's a strategy:
 - (a) Prove that $A\mathbf{x} \cdot \mathbf{y} = -\mathbf{x} \cdot A\mathbf{y}$ for any vectors \mathbf{x} and \mathbf{y} , where \cdot is the complex dot product $\mathbf{x} \cdot \mathbf{y} = \bar{\mathbf{x}}^T \mathbf{y}$.
 - (b) Suppose that \mathbf{x} is an eigenvector with eigenvalue λ . Plug this in for \mathbf{y} in the above and conclude that $\bar{\lambda} = -\lambda$.
 4. Suppose that A is a symmetric matrix, and all of its eigenvalues are equal. Prove that A is a multiple of the identity matrix.
 5. Let A be the matrix $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$.
 - (a) Explain why \mathbf{x} is positive-definite by writing $\mathbf{x}^T A \mathbf{x}$ as a sum of two squares, where $\mathbf{x} = (x, y)$.
 - (b) Sketch the ellipse defined by $\mathbf{x}^T A \mathbf{x} = 1$.
 6. Let $A = \begin{pmatrix} x & 2 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & x \end{pmatrix}$. For what value(s) of x is this matrix diagonalizable by an orthogonal matrix? for what value(s) of x is it positive definite?
 7. Two masses are arranged in a "fixed-fixed" spring system, with spring constants 1, 2, and 3. Write down the equation for the masses to be in equilibrium. If they start out of equilibrium, what equation governs their motion? Do we know a strategy to solve an equation of this form?