Math 18.06, r3/5
Problems #11
May 14, 2013

1. Suppose $\lambda_1 = 2$ and $\lambda_2 = 5$ and $x_1 = (1,1,1)$ and $x_2 = (1,-2,1)$. Choose $\lambda_3$ and $x_3$ so that $A$ is symmetric positive semidefinite but not positive definite.

2. Suppose: $A$ is positive definite symmetric
   $Q$ is orthogonal (same size as $A$)
   $B$ is $Q^T AQ = Q^{-1}AQ$.

   Show that: $B$ is also symmetric.
   $B$ is also positive definite.

3. Which of the following are linear transformations? Why or why not?

   (a) $T : \mathbb{R}^3 \to \mathbb{R}$ defined by $T(\mathbf{v}) = |\mathbf{v}|$

   (b) $T(\mathbf{v}) =$ largest component of $\mathbf{v}$

   (c) $T(a,b,c) = (b,c,a)$.

4. Consider the matrix with SVD

$$A = U\Sigma V^T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

   Sketch the image of the square with vertices $(0,0)$, $(1,0)$, $(0,1)$, and $(1,1)$ under $A$ by applying each of the three matrices of the SVD.

5. Consider the vector space of functions spanned by $\sin x$, $\cos x$, $\sin 2x$, and $\cos 2x$.

   (a) Write down the matrices $\partial$ and $\int$ for differentiation and integration on this space.

   (b) What are $\partial \int$ and $\int \partial$?

   (c) How can you find the matrix for taking the second derivative?

   (d) How would your answer change if we added the function 1 to this basis?

6. Suppose that $f$ has period $2\pi$ and $f(-x) = -f(x)$ for all $x$ (i.e. $x$ is an odd function). What does this tell you about its Fourier coefficients?

7. Why are the eigenvalues of any Hermitian matrix real?

8. What class of matrices does $P$ belong to? (invertible, Hermitian, unitary)

$$P = \begin{bmatrix} 0 & i & 0 \\ 0 & 0 & i \\ i & 0 & 0 \end{bmatrix}$$

   What are the eigenvalues of $P$?