Announcements

- MML due tonight
- Written homework due Tues
- If free-trial doesn't convert, call Pearson tech support.
- First quiz will be next week.

Today: 5.1 & 5.2.

Math 121 (Lesieutre); §5.1 & 5.2; September 8, 2017

1. A **function** is a relation that maps each element, x, of the domain to exactly **one** element, y, of the range. You can check if a relation is a function using the vertical line test. We can also define **composition of functions** using:

$$(f \circ g)(x) = f(g(x)).$$

2. Given $f(x) = x^2 - 3x$ and $g(x) = x + 1$, find the following.

a) What is the domain of each function?

all real numbers.

$g(f(x)) = \underbrace{(x^2 - 3x)}_{f} + 1.$

$= x^2 - 3x + 1$

b) What are $f(g(x))$ and $g(f(x))$?

plug in $g(x)$ to all the x's in formula for f.

$f(g(x)) = (x+1)^2 - 3(x+1) = (x^2+2x+1) - (3x+3) = x^2 - x - 2.$

c) What is the domain of your answers in part (b)?

all real numbers.

d) What is $f(g(-1))$? $f(g(-1)) = g(-1)$ plugged into f.

$g(-1) = (-1) + 1 = 0$

$f(g(-1)) = f(0) = 0^2 \cdot 3 \cdot 0 = 0.$

3. Sometimes you have to think a little harder about the domain of a composed function: you need to restrict to x's for which the output of the inner function is a valid input to the second function. For the pairs below, find the composition $(f \circ g)(x)$, and determine its domain.

domain of f: $[-1, \infty)$

i.e. $x \geq -1$.

domain of g: all real #'s.

The domain of $f(g(x))$ is all x in domain of g such that $g(x)$ is in the domain of f.

a) $f(x) = \sqrt{x+1}$, $g(x) = 3x$.

$f(g(x)) = \sqrt{3x+1}.$

b) $f(x) = \dfrac{x}{x+3}$, $g(x) = \dfrac{2}{x}$.

$\boxed{\{x \mid x \geq -\frac{1}{3}\}}$

Note: $x = -1$ is in domain of $g(x)$.

domain of $f(g(x))$:

Domain of $f(g(x))$ is

$3x + 1 \geq 0$

$3x \geq -1 \Rightarrow \boxed{x \geq -\frac{1}{3}}$ $\boxed{[-\frac{1}{3}, \infty)}$

$f(x) = \frac{x}{x+3}, \quad g(x) = \frac{2}{x}$

$$f(g(x)) = \frac{g(x)}{g(x)+3} = \frac{\frac{2}{x}}{\left(\frac{2}{x}+3\right)} = \frac{2/x}{\left(\frac{2+3x}{x}\right)}$$

$$= \left(\frac{2}{x}\right)\cdot\left(\frac{x}{2+3x}\right) = \frac{2}{2+3x}.$$

Domain: $2+3x=0$

$3x=-2$

$\boxed{x=-\frac{2}{3}}$ not in domain.

$x=0$ not in domain:

~~we~~ we can't plug in to g!

Domain: $\{x \mid x\neq 0, x\neq -\frac{2}{3}\}$.

↑

you need to be able to plug x into g(x) (rules out 0)

and then plug in g(x) to f. (rules out $-2/3$).

4. Now let's *de*compose functions. For the H's given, find f and g such that $f \circ g = H$.

$H(x) = (2x+3)^7,$	$H(x) = \sqrt{1-x^2},$	$H(x) = \dfrac{1}{3x^3-1}.$
what you do second $\rightarrow$ $f(x)=x^7$	$f(x)=\sqrt{1-x}$	$f(x)=1/x$
what you do first $\rightarrow$ $g(x)=2x+3$	$g(x)=x^2$	$g(x)=3x^3-1$
	or	
	$f(x)=\sqrt{x}$	
	$g(x)=1-x^2$	

5. What is an inverse function?

6. Let $f(x) = x^2 + 4$ with domain $x \geq 0$. Find f^{-1}. (Why was the domain restricted?) Sketch a graph of both.

7. Show that the following are inverse functions by showing $f(g(x)) = x = g(f(x))$.

$$f(x) = x^3 - 8, \qquad g(x) = \sqrt[3]{x+8}$$

8. Given the rational function $f(x) = \dfrac{2x+3}{x+2}$, answer the following.

a) What is the domain of f? Sketch a graph.

b) Find f^{-1} and sketch the graph.

4. Now let's *de*compose functions. For the H's given, find f and g such that $f \circ g = H$.

$$H(x) = (2x+3)^7, \qquad H(x) = \sqrt{1-x^2}, \qquad H(x) = \frac{1}{3x^3-1}.$$

You can only take inverse of a one-to-one function: this means different inputs give different outputs.

5. What is an inverse function?

An inverse function for $f(x)$ is a function $f^{-1}(x)$ such that $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$.

"A function that undoes f"

6. Let $f(x) = x^2 + 4$ with domain $x \geq 0$. Find f^{-1}. (Why was the domain restricted?) Sketch a graph of both.

$f(x) = x^2 + 4$. Rule for finding inverse:

1) write $y = f(x)$ 3) solve for y

2) swap x & y

$y = x^2 + 4$

$x = y^2 + 4$

$x - 4 = y^2$

$y = \sqrt{x-4}$.

$x \geq 0$ restriction makes it one-to-one.

7. Show that the following are inverse functions by showing $f(g(x)) = x = g(f(x))$.

$$f(x) = x^3 - 8, \qquad g(x) = \sqrt[3]{x+8}$$

$f(g(x)) = (\sqrt[3]{x+8})^3 - 8 = (x+8) - 8 = x.$

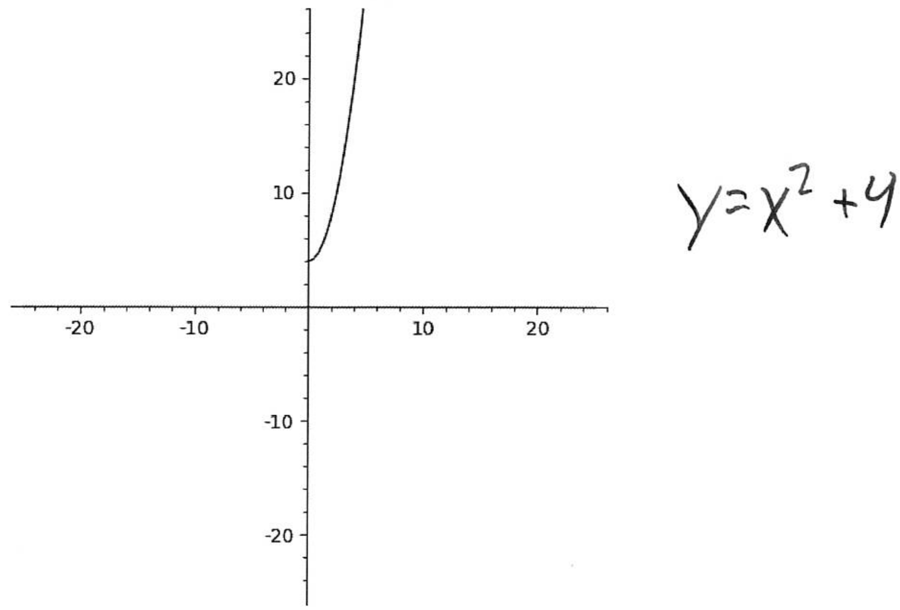
$g(f(x)) = \sqrt[3]{(x^3-8)+8} = \sqrt[3]{x^3} = x.$ ✓

8. Given the rational function $f(x) = \dfrac{2x+3}{x+2}$, answer the following.

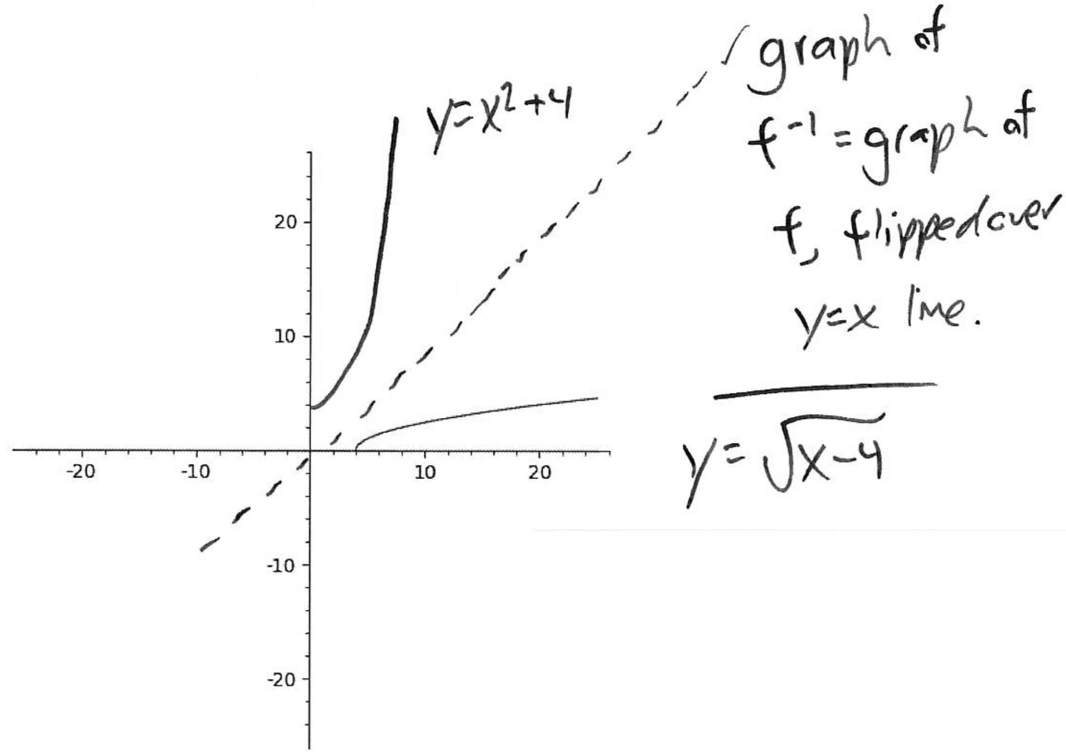
a) What is the domain of f? Sketch a graph.

b) Find f^{-1} and sketch the graph.

$$y = \frac{3-2x}{x-2}$$



And the inverse:



You probably notice something odd: the graph of the inverse function is the same of the graph of the original function, but reflected over the line $y = x$.