Math 121 (Lesieutre); §4.3; September 6, 2017

The steps for graphing a rational function:

1. Factor the numerator and denominator, and write the function in lowest terms.
2. Set the numerator equal to zero to find the x-intercepts (don't forget about multiplicity!)
3. Plug in $x = 0$ to find the y-intercept.
4. Set the denominator equal to zero to find the vertical asymptotes.
5. Find the horizontal asymptotes, depending on the degree of numerator and denominator:
   (a) If degree of numerator is less than degree of denominator, $y = 0$ is asymptote.
   (b) If degree of num. = degree of denom., then $y = \dfrac{\text{leading coef of numerator}}{\text{leading coef of denominator}}$ is asymptote.
   (c) If degree of numerator is greater than degree of denominator, there is no horizontal asymptote.
6. Split the x-axis into intervals, breaking it up whereever the numerator or denominator is 0. For each interval, figure out if the graph is above or below the axis in that interval, by plugging in a test number.

1. $f(x) = \dfrac{x^2 + x - 12}{x^2 - 4}$

2. $g(x) = \dfrac{2x^2 + 2x - 24}{x^2 - x - 6}$ (there's a cancellation – so what?)

3. $h(x) = \dfrac{x}{x^2 - 4}$

4. $i(x) = \dfrac{x^2 + 3x + 2}{x - 1}$ (there's a slant asymptote – can you figure out what it is?)

5. Given the graph below, find all asymptotes, intercepts, end behavior, domain, and range. Then use this to find a possible formula for the rational function.