Math 121 (Lesieutre); §5.1 & 5.2; September 8, 2017

1. A **function** is a relation that maps each element, x, of the domain to exactly **one** element, y, of the range. You can check if a relation is a function using the vertical line test. We can also define **composition of functions** using:

$$(f \circ g)(x) = f(g(x)).$$

2. Given $f(x) = x^2 - 3x$ and $g(x) = x + 1$, find the following.

a) What is the domain of each function?

b) What are $f(g(x))$ and $g(f(x))$?

c) What is the domain of your answers in part (b)?

d) What is $f(g(-1))$?

3. Sometimes you have to think a little harder about the domain of a composed function: you need to restrict to x's for which the output of the inner function is a valid input to the second function. For the pairs below, find the composition $(f \circ g)(x)$, and determine its domain.

a) $f(x) = \sqrt{x+1}$, $g(x) = 3x$.

b) $f(x) = \dfrac{x}{x+3}, \quad g(x) = \dfrac{2}{x}$.

4. Now let's *de*compose functions. For the H's given, find f and g such that $f \circ g = H$.

$$H(x) = (2x+3)^7, \qquad H(x) = \sqrt{1-x^2}, \qquad H(x) = \frac{1}{3x^3-1}.$$

5. What is an inverse function?

6. Let $f(x) = x^2 + 4$ with domain $x \geq 0$. Find f^{-1}. (Why was the domain restricted?) Sketch a graph of both.

7. Show that the following are inverse functions by showing $f(g(x)) = x = g(f(x))$.

$$f(x) = x^3 - 8, \qquad g(x) = \sqrt[3]{x+8}$$

8. Given the rational function $f(x) = \dfrac{2x+3}{x+2}$, answer the following.

a) What is the domain of f? Sketch a graph.

b) Find f^{-1} and sketch the graph.