Math 121 (Lesieutre); §4.1, Part 1; August 28, 2017

1. *Here are three polynomials. For each one: what are the degree, the coefficients, the constant term, the leading coefficient, and the leading term?*

$$p(x) = 5x^7 - 3x^3 + 2x - 11, \quad q(x) = -12 + 4x^6 - 2x^2 + 8x^9, \quad r(x) = x(x+2)(x-1).$$

For the first, $p(x)$, the degree is 7 (that's the biggest power of x), the coefficients are 5, -3, 2, and -11 (more specifically: the x^7 coefficient is 5, etc.) The constant term is the one with no x's: -11. The leading coefficient is the coefficient on the x^7 term, which is 5. THe leading term is $5x^7$.

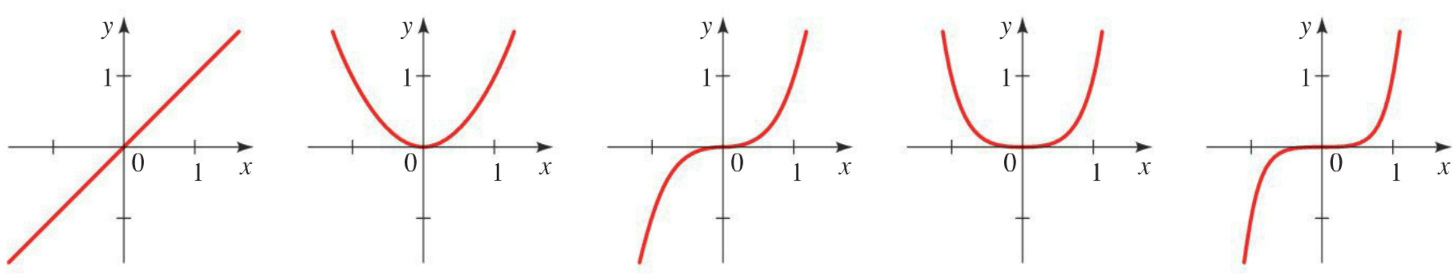
Next, $q(x)$. The terms aren't in order this time, so we need to be a little bit careful. The degree is 9, which is the largest exponent on an x. The coefficients are -12, 4, -2, and 8. The constant term is -12. The leading term is $8x^9$: that's the largest power of x, which is 9. The leading coefficient is 8, since that's the number in front of the x^9 term.

$r(x)$ is a little tricky, because this one is already factored. To see the coefficients, it's probably helpful to multiply it out:

$$x(x+2)(x-1) = (x^2+2x)(x-1) = x^3 - x^2 + 2x^2 - 2x = x^3 + x^2 - 2x.$$

Now we can see: the degree is 3, the coefficients are 1, 1, -2, and 0. The leading term is x^3, and the leading coefficient is 1. Here the constant term is 0: everything has an x. You could have figured out much of that – maybe the degree, the constant term, the leading term, and the leading coefficient – without actually doing the full expansion, but to see all the coefficients you don't have much choice but to multily.

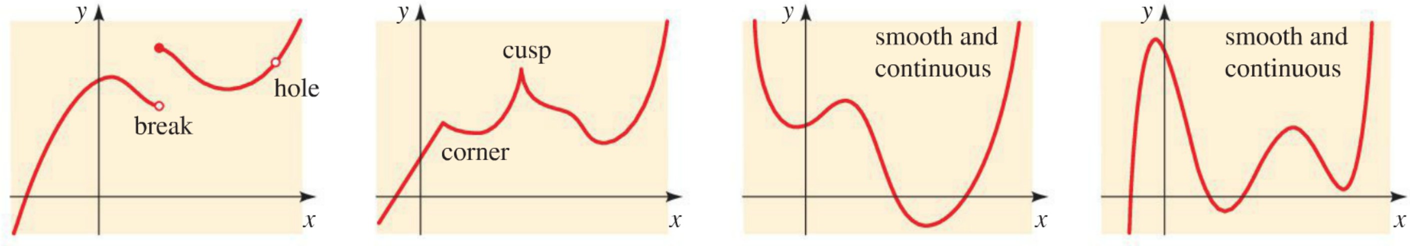
2. *Some basic examples:*



(a) $y = x$ (b) $y = x^2$ (c) $y = x^3$ (d) $y = x^4$ (e) $y = x^5$

These are all "power functions": there's only one term. Note that when the exponent is even it goes in the positive directions on both sides, while when it's negative, it goes positive in the positive x direction and negative in the negative x direction. (Do you see why this should be?)

3. *Polynomials are smooth functions:*

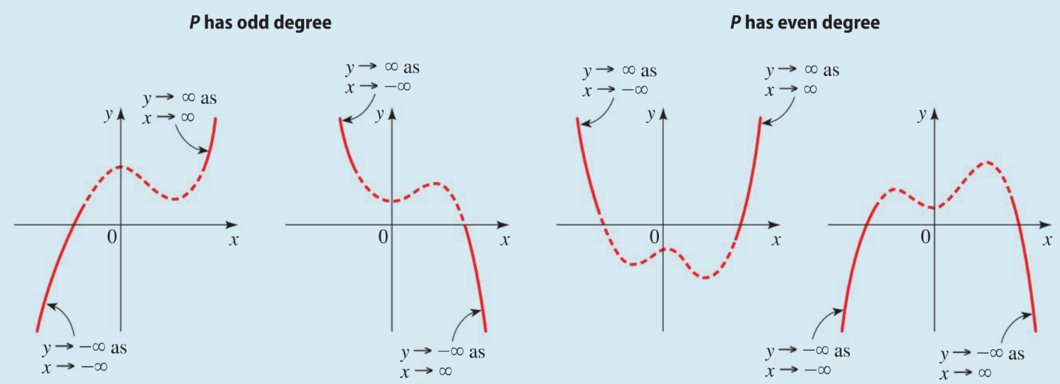


Not the graph of a polynomial function | Not the graph of a polynomial function | Graph of a polynomial function | Graph of a polynomial function

This basically means you can draw them without lifting your pencil, and there are no “kinks” where the graph suddenly changes direction.

4. *End behavior and general shape:*

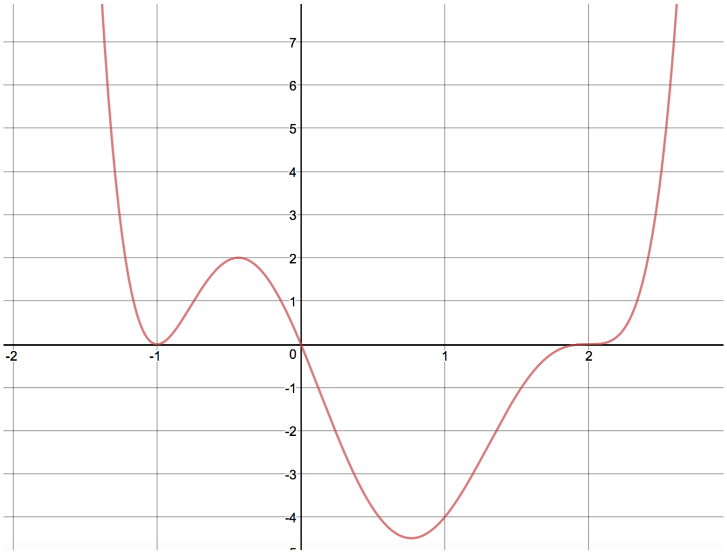
The end behavior of the polynomial $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ is determined by the degree n and the sign of the leading coefficient a_n, as indicated in the following graphs.



Leading coefficient positive | Leading coefficient negative | Leading coefficient positive | Leading coefficient negative

Maybe it helps to remember what's going on for parabolas, if you've learned that before. $y = (positive)x^2$ opens up, and $y = (negative)x^2$ opens down – the x term doesn't matter. This is what happens for any polynomial with even degree.

5. *Zeroes and multiplicity:*



The polynomial you see here has a zero at $x = -1$ with multiplicity 2, $x = 0$ with multiplicity 1, and at $x = 2$ with what looks like multiplicity 3 (one can't really be sure from the graph, most of the time).

One important thing to remember: if the multiplicity is odd, the graph will cross from one side of the x-axis to the other. If it's even, the function will be tangent to the axis: it "bounces off".

Our function could be something like

$$f(x) = (x+1)^2(x)(x-2)^3.$$

6. *Let's say we have a polynomial with zeros $x = -2$, $x = 0$, and $x = 2$, and it passes through the point $(-4, 16)$. Can you come up with a formula? Sketch the graph?*

Let's start with a guess:

$$f(x) = (x+2)(x)(x-2)$$

This is close: it has the right zeroes. The problem is that if $x = -4$, then $y = (-2)(-4)(-6) = -48$, which isn't what we want. However: we can multiply our equation by a constant without changing the zeroes. So our next guess would be $c(x+2)(x)(x-2)$, where c is some as-yet-undetermined constant. How to find c? Well, if $x = -4$, then $y = -48c$. To get $c = 16$, we need to use $c = -1/3$. That gives the final answer:

$$g(x) = -\frac{1}{3}(x+2)(x)(x-2).$$