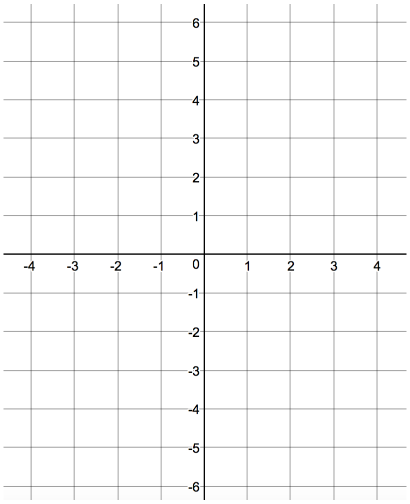
Math 121 (Lesieutre); §4.1: Polynomials, part 2; August 30, 2017

1. *Given the following information about a* 5^{th} *degree polynomial, sketch a graph and find its equation.* $f(-2) = 0$, $f(-1) = 0$, $f(1) = 0$, $f(2) = 0$, *and* $f(0) = -1$, *and the zero at* $x = -1$ *has multiplicity two.*



The information about the zeros tells us that the polynomial must look like this:

$$f(x) = a(x+2)(x+1)^2(x-1)(x-2)$$

(the squared is there on the $x + 1$ to make this root have multiplicity 2).

Here a the leading coefficient, which we still need to figure out. To do that, we use the information that $f(0) = -1$. Plugging in 0 to the equation gives

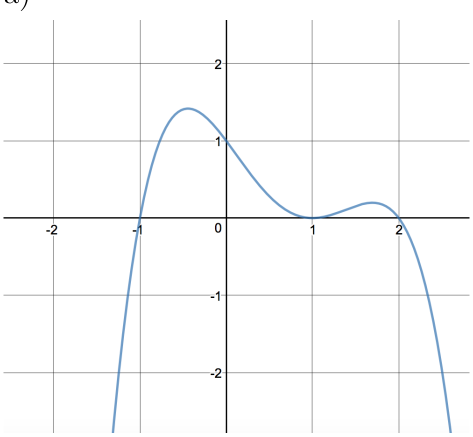
$$f(0) = a(0+2)(0+1)^2(0-1)(0-2) = a \cdot 2 \cdot 1 \cdot (-1) \cdot (-2) = 4a.$$

We want $f(0) = -1$, so we need to set $4a = -1$, which gives $a = -\frac{1}{4}$. That gives our final answer:

$$f(x) = -\frac{1}{4}(x+2)(x+1)^2(x-1)(x-2).$$

2. *Given the following graphs of polynomials, find the function that matches the graph. First, list all zeros with multiplicity, then use another point on the graph to find the leading coefficient.*

a)



The zeroes are $x = -1$, $x = 1$, and $x = 2$. It looks to me like the multiplicity at $x = -1$ is 1, at $x = 1$ is 2 (it "bounces off the axis"), and at $x = 2$ is 1. So our polynomial is going to look like

$$f(x) = a(x+1)(x-1)^2(x-2),$$

where again a is the leading coefficient that we need to figure out.

From the graph, we also see that $f(0) = 1$: let's use that information to figure out what polynomial this is.

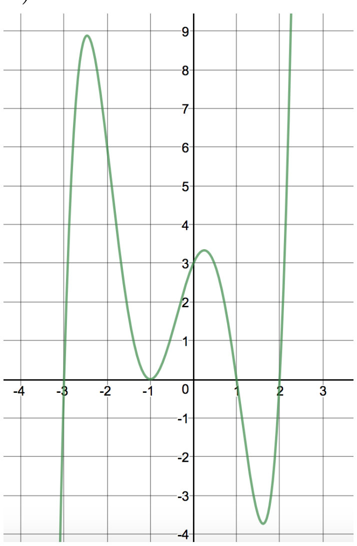
As in the previous problem, we can plug in $x = 0$ and use that to solve for a.

$$f(0) = a(0+1)(0-1)^2(0-2) = a \cdot 1 \cdot (-1)^2 \cdot (-2) = -2a.$$

So $-2a = 1$, which means that $a = -\frac{1}{2}$. Our final polynomial is therefore

$$f(x) = -\frac{1}{2}(x+1)(x-1)^2(x-2).$$

b)



This is kind of a crazy-looking polynomial, but we can use the same basic procedure. The roots are going to be $x = -3$ (multiplicity 1), $x = -1$ (multiplicity 2), $x = 1$ (multiplicity 1), $x = 2$ (multiplicity 1). So it will be

$$f(x) = a(x+3)(x+1)^2(x-1)(x-2).$$

But we still need to know: what is a? From the graph, it looks like $f(0) = 3$, which we can use to solve for a.

$$f(0) = a(0+3)(0+1)^2(0-1)(0-2) = a \cdot 3 \cdot 1 \cdot (-1) \cdot (-2) = 6a,$$

whence $6a = 3$, so $a = \frac{1}{2}$.

Our final answer is

$$f(x) = \frac{1}{2}(x+3)(x+1)^2(x-1)(x-2).$$

3. *Answer the following about each polynomial below.*

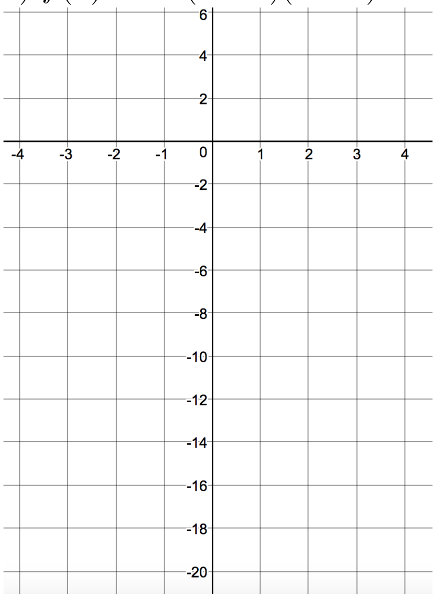
a) *Determine the end behavior: what happens to $f(x)$ as $x \to \infty$? As $x \to -\infty$?*

b) *Find all x and y intercepts of the function.*

c) *Determine the zeros and their multiplicities. What do the graph look like at these points?*

d) *Use your answers to sketch a graph of the polynomial.*

a) $f(x) = -2(x+2)(x-2)^2$

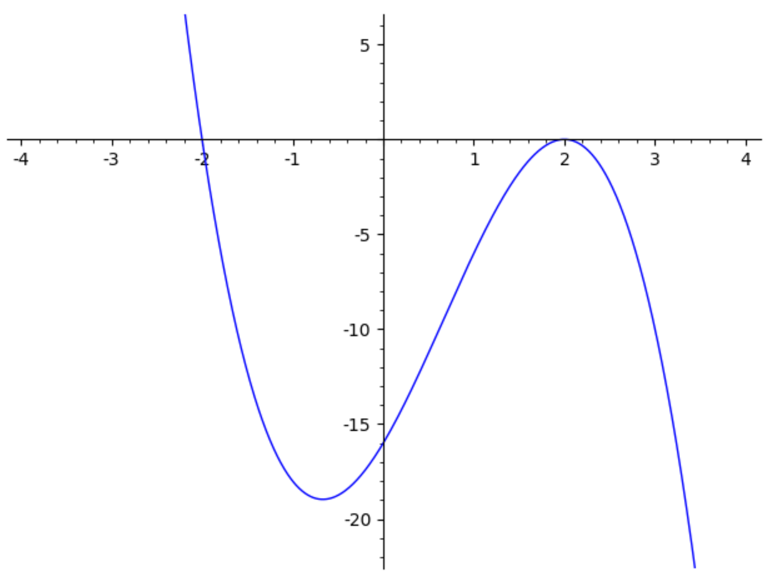


First, we need to figure out the degree and leading coefficient. The degree is 3: when we multiply out, we won't get any terms bigger than x^3. The leading coefficient is -2. The degree is odd and the leading coefficient is negative, so as $x \to \infty$ we get $f(x) \to -\infty$, while as $x \to -\infty$ we get $f(x) \to \infty$.

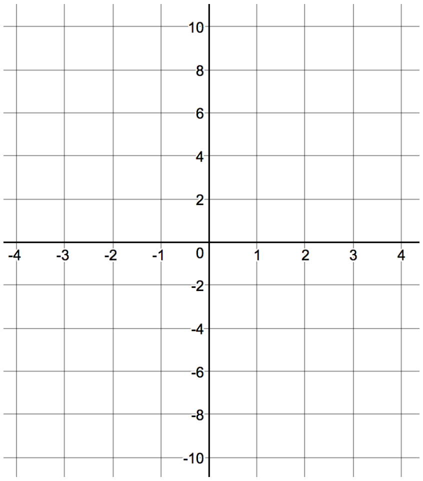
We have $f(0) = -2(0+2)(0-2)^2 = -16$, so the y-intercept is the point $(0, -16)$ (that's why you're given such lopsided graph paper).

The x-intercepts are the same thing as roots, which occur at $x = -2$ and $x = 2$: the points are $(-2, 0)$ and $(2, 0)$. The first has multiplicity 1, while the second has multiplicity 2. The graph near the first will cross the axis, while at the second it will look like a parabola that's tangent to the axis.

Here's a computer plot.



b) $g(x) = x^3(x^2 - 4)$



This is a tough one. The degree is 5 and the leading coefficient is 1, and so as $x \to \infty$ we get $f(x) \to \infty$, while when $x \to -\infty$ we get $f(x) \to -\infty$.

The y-intercept is the point $(0, 0)$, since $f(0) = 0$.

To see the roots, we want to factor it a little further: $g(x) = x^3(x-2)(x+2)$. This polynomial has roots at $x = 0$ (mult 3), $x = 2$ (mult 1), and $x = -2$ (mult 1). The roots are -2 (mult 1) and 2 (mult 2).

So, coming from the left, the graph rises up from $-\infty$, crosses the axis at $x = -2$, and then crosses it again at $x = 0$ (the multiplicity is odd, so it crosses – however, near $x = 0$, the graph will level out as it crosses, because the multiplicity is 3). It is now under the axis, but crosses back above at $x = 2$. Here's an accurate plot.