Math 121 (Lesieutre); §5.3; September 13, 2017

1. *Remember that*

$$\log_a x = y \quad \Longleftrightarrow \quad a^y = x$$

*When you see "*log*" without an a subscript, it means* $\log_{10}$*. When you see* ln*, it means* $\log_e$*. Use this to evaluate the following logarithmic expressions:*

$\log_2 8,\quad \log_5 125,\quad \log_3 81,\quad \log_{16} 4,\quad \log 0.01,\quad \ln\left(\frac{1}{\sqrt{e}}\right)$

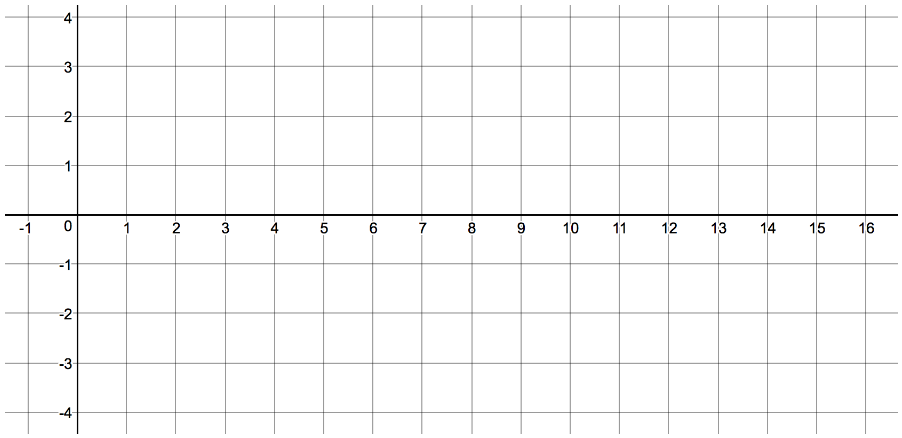
$$\begin{aligned}
\log_2 8 &= 3, \text{ because } 2^3 = 8 \\
\log_5 125 &= 3, \text{ because } 5^3 = 125 \\
\log_3 81 &= 4, \text{ because } 3^4 = 81 \\
\log 164 &= \frac{1}{2}, \text{ because } 16^{1/2} = \sqrt{16} = 4 \\
\log 0.01 &= -2, \text{ because } 10^{-2} = 0.01 \\
\ln\left(\frac{1}{\sqrt{e}}\right) &= -\frac{1}{2}, \text{ because } e^{-1/2} = 1/\sqrt{e}.
\end{aligned}$$

2. *Evaluate:*

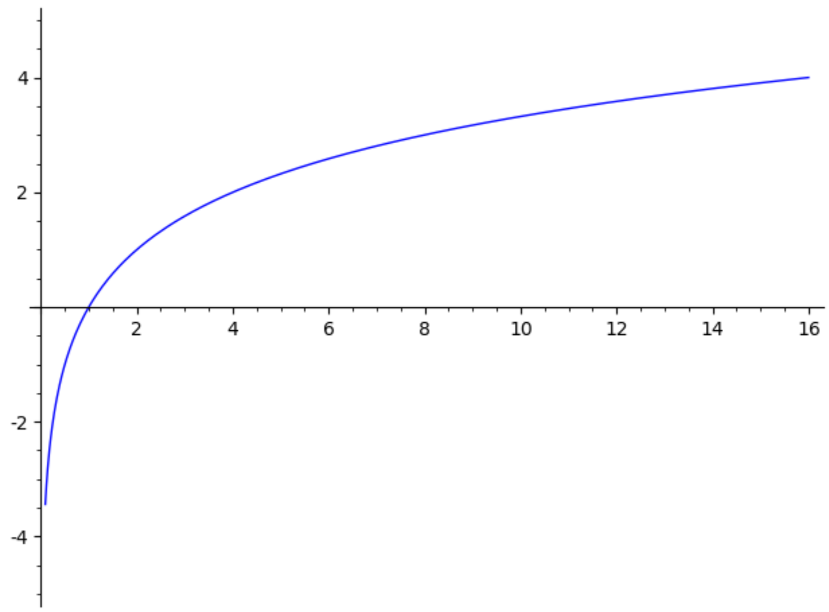
$\log_a 1,\quad \log_a a,\quad \log 10^{12},\quad \log_a a^x,\quad 10^{\log 1000},\quad a^{\log_a x}$

$$\begin{aligned}
\log_a 1 &= 0 \\
\log_a a &= 1 \\
\log 10^{12} &= 12 \\
\log_a a^x &= x \\
10^{\log 1000} &= 1000 \\
a^{\log_a x} &= x
\end{aligned}$$

3. *Sketch the graph of* $\log_2(x)$ *by plotting some points and interpolating. What are the domain and range, and are there any asymptotes?*

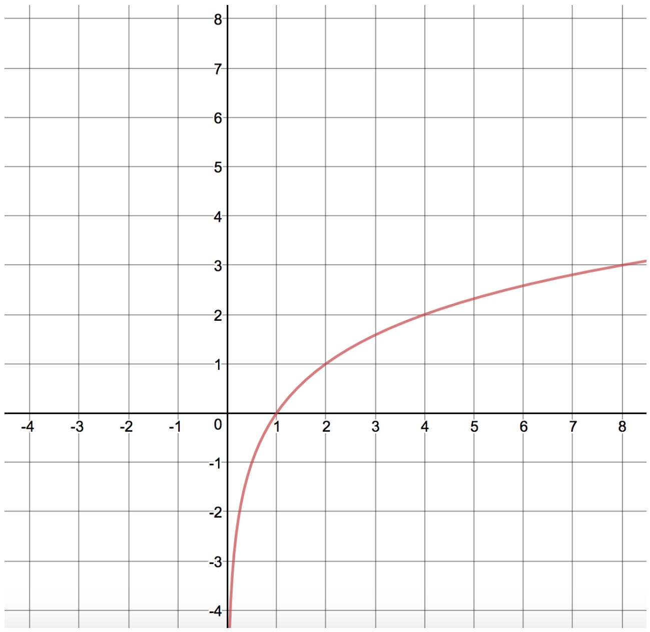


Here's the graph:

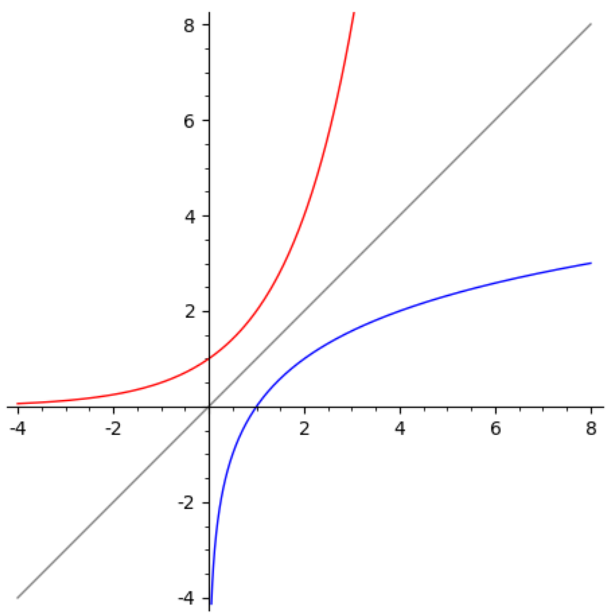


The domain is all positive real numbers. The range is all real numbers. There's a vertical asymptote at $x = 0$ (notice that it looks a little different than the ones we encountered with rational functions – the domain doesn't include negative numbers, so it doesn't come out again on the other side).

4. *What is the inverse function of* $f(x) = \log_2 x$*?* f *is sketched below, sketch* $f^{-1}(x)$ *on the same plane.*



Remember our rule for finding inverse functions. We write $y = \log_2 x$. Then we switch the x and y: $x = \log_2 y$. Then we solve for y, giving $y = 2^x$. Here's a graph showing both:



As always, the function and its inverse differ by flipping over the axis.

5. *To solve logarithmic equations, rewrite them using exponentials.* $\log_2(2x+1) = 3, \quad \log(5x+80) = 3, \quad \log_6 36 = 5x+3.$

First:

$$\begin{aligned}\log_2(2x+1) &= 3\\ 2x+1 &= 2^3 = 8\\ \\ 2x &= 7\\ x &= \frac{7}{2}.\end{aligned}$$

Second:

$$\begin{aligned}\log(5x+80) &= 3\\ 5x+80 &= 10^3 = 1000\\ 5x &= 920\\ x &= 184.\end{aligned}$$

At last:

$$\begin{aligned}\log_6 36 &= 5x+3\\ 2 &= 5x+3\\ -1 &= 5x\\ x &= -\frac{1}{5}.\end{aligned}$$

6. $\log_5(x^2+4x+4) = 2, \quad \ln e^{-2x} = 8$

First:

$$\begin{aligned}\log_5(x^2+4x+4) &= 2\\ x^2+4x+4 &= 5^2 = 25\\ x^2+4x-21 &= 0\\ (x-3)(x+7) &= 0.\end{aligned}$$

So $x = 3$ or $x = -7$.

Second:

$$\begin{aligned}\ln e^{-2x} &= 8\\ -2x &= 8\\ x &= -4.\end{aligned}$$

7. *We can also use logarithms to solve exponential equations.* $e^{3x} = 10$, $2e^{2x+5} = 16$, $5^{3x-1} = 11$, $2 \cdot 10^{x-2} = 5$

$$\begin{aligned} e^{3x} &= 10 \\ 3x &= \ln 10 \\ x &= \frac{1}{3}\ln 10 \end{aligned}$$

$$\begin{aligned} 2e^{2x+5} &= 16 \\ e^{2x+5} &= 8 \\ 2x+5 &= \ln 8 \\ 2x &= \ln 8 - 5 \\ x &= \frac{1}{2}(\ln 8 - 5) \end{aligned}$$

$$\begin{aligned} 5^{3x-1} &= 11 \\ 3x-1 &= \log_5 11 \\ x &= \frac{1}{3}(\log_5 11 + 1) \end{aligned}$$

$$\begin{aligned} 2 \cdot 10^{x-2} &= 5 \\ 10^{x-2} &= \frac{5}{2} \\ x-2 &= \log\left(\frac{5}{2}\right) \\ x &= 2 + \log\left(\frac{5}{2}\right) \end{aligned}$$