Math 210 (Lesieutre)
11.6: Calculus of vector valued functions
January 23, 2016

Problem 1. The position of a planet is given by $\mathbf{r}(t) = \langle 3\cos t, 2\sin t\rangle$. Find a unit tangent vector to the path of the planet.

Problem 2. Consider the function $\mathbf{r}(t) = \langle \cos t, \sin t, t\rangle$

a) Sketch $\mathbf{r}(t)$ for $0 \leq t \leq 4\pi$.

b) Compute $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$. Plug in $t = 0$ and $t = \pi/2$ to your answer. Do these make sense? Can you think of any other "sanity checks"?

c) Find a unit tangent vector for the parametrized curve, in terms of t.

Problem 3. Consider the path given by $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$.

a) Write down an equation for the tangent line to $\mathbf{r}(t)$ at $t = 1$.

b) What is $\frac{d}{dt}(\mathbf{r}(t) \cdot \langle 1, 2, 3 \rangle)$?

Problem 4. a) What is

$$\frac{d}{dt} |\mathbf{x}(t)|^2,$$

in terms of $\mathbf{x}(t)$ and its derivatives?

b) Check your answer to (a) for $\mathbf{x}(t) = \langle \cos t, \sin t \rangle$. Does your answer make sense?

c) Suppose that $\mathbf{x}(t)$ has constant length. What does this tell you about the relationship between $\mathbf{x}(t)$ and $\mathbf{x}'(t)$?

Problem 5. Let $\mathbf{s}(t) = \langle -\sin t, \cos t, 1 \rangle$. Compute $\int \mathbf{s}(t)\, dt$.