

Math 210 (Lesieutre)
12.3: Limits and continuity
February 6, 2016

Problem 1. a) Evaluate the limit:

$$\lim_{(x,y) \rightarrow (1,2)} 2x^2 + \sqrt{xy}$$

b) Evaluate the limit:

$$\lim_{(x,y) \rightarrow (3,0)} \frac{3x^2 + y}{x + y}$$

Problem 2. Evaluate the following limits along the path $y = mx$. Does the limit exist?

a)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^4 + y^4}{x^2 + y^2}$$

b)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{(x + y)^2}{x^2 + y^2}$$

Problem 3. a) Give an example of a function $f(x, y)$ that is *not* continuous at $(1, 2)$. Can you come up with an example that is continuous for all (x, y) other than $(1, 2)$?

b) Give an example of a function $g(x, y)$ that is *not* continuous for any (x, y) satisfying $y = x^2$.

Problem 4. Is the function

$$p(x, y) = \begin{cases} \frac{x^2 y^2}{x^4 + y^4} & \text{if } (x, y) \neq (0, 0), \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$$

continuous at $(0, 0)$?

Problem 5. Set up the integral for the arc length of $\mathbf{r}(t) = \langle \cos t, \sin t \rangle$ for $0 \leq t \leq \pi$.