Math 210 (Lesieutre)
12.6: Directional derivatives and the gradient
February 10, 2017

Problem 1. Suppose that $f(x, y) = xy^2$, and $x(s, t) = 2s + t$ and $y(s, t) = s \cos t$. Compute the partial derivative $\frac{\partial f}{\partial s}$.

Problem 2. Consider the ellipse defined by $F(x, y) = 0$, where $F(x, y) = x^2 + xy + y^2 - 1$. Compute $\frac{dy}{dx}$.

Problem 3. Consider the function $f(x, y) = 10 - x^2 - 4y^2$.

a) Compute the gradient $\nabla f(x, y)$.

b) Find the derivative in the direction of the vector $\mathbf{v} = \langle 1, 1 \rangle$ at the point $(1, 1)$. (Watch out! This isn't a unit vector.)

c) Find the directional derivative in the direction of the vector $\mathbf{u} = \langle 0, -1 \rangle$ at $(1, 1)$.

d) Sketch some level curves of $f(x, y)$, including the level curve with $z = 5$.

e) Find the unit vectors in the directions of steepest ascent and descent at the point $(1, 1)$. Do your answers make sense?

f) Find the directional derivative in the direction of steepest ascent. Is this steeper than the answers you got for directional derivatives earlier in the problem?

g) Find a direction that is tangent to the level curve $z = 5$ at the point $(1, 1)$. What is the directional derivative in this direction?

Problem 4. Suppose that a function has gradient $\nabla f(0, 0) = (1, 1)$.

a) What is the directional derivative of this function in a direction with angle θ?

b) Plot the directional derivative for $0 \leq \theta \leq 2\pi$. For what θ is it maximized? Zero?