Math 210  (Lesieutre)
12.6 Directional derivatives, and some review
February 13, 2017

**Problem 1.** Suppose that a function has gradient $\nabla f(0,0) = (1,1)$.

a) What is the directional derivative of this function in a direction with angle $\theta$?

b) Plot the directional derivative for $0 \le \theta \le 2\pi$. For what $\theta$ is it maximized? Zero?

**Problem 2.** Consider the function $f(x,y) = 10 - x^2 - 4y^2$.

a) Sketch some level curves of $f(x,y)$, including the level curve with $z = 5$.

b) Find the unit vectors in the directions of steepest ascent and descent at the point $(1,1)$. Do your answers make sense?

c) Find the directional derivative in the direction of steepest ascent. Is this steeper than the answers you got for directional derivatives earlier in the problem?

d) Find a direction that is tangent to the level curve $z = 5$ at the point $(1,1)$. What is the directional derivative in this direction?

**Problem 3.** Find the parametrization for a line which...

a) Has $\mathbf{r}(0) = (1, 2, 3)$ and $\mathbf{r}(1) = (1, -2, 1)$.

b) Normal to the plane $3x - 2y + z = 0$ and passes through the origin.

c) The intersection of the planes $x + y + z = 3$ and $x - y + 2z = 1$.

d) Tangent to the curve $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ at $t = 2$.

**Problem 4.** Consider the two vectors $\mathbf{u} = \langle 1, 2, 3 \rangle$ and $\mathbf{v} = \langle -1, -1, -1 \rangle$.

a) What is the angle between $\mathbf{u}$ and $\mathbf{v}$?

b) If a triangle has $(0, 0, 0)$ as a vertex, with $\mathbf{u}$ and $\mathbf{v}$ the two edges from this vertex, what is the vector for the third edge?

c) What is the area of this triangle?