

Math 210 (Lesieutre)

12.7: Tangent planes and linear approximation

February 17, 2017

Problem 1. Let's start with a tangent plane to a sphere.

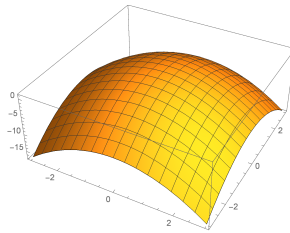
a) Write down the equation $f(x, y, z)$ for a sphere with center $(1, 2, 3)$ and radius 3.

b) One point on the sphere is $(a, b, c) = (2, 0, 5)$. Compute the gradient ∇f , and evaluate $\nabla f(a, b, c)$.

c) Use your answer to write down the equation for the tangent plane to the sphere at (a, b, c) .

d) Try to plot the sphere and the plane and convince yourself that this answer is reasonable.

Problem 2. Let $f(x, y) = 1 - x^2 - y^2$.



a) Find the tangent plane to the graph at $(x, y) = (0, 0)$. Does your answer make sense?

b) Find the tangent plane to the graph at $(x, y) = (1, -1)$. Does your answer make sense?

Problem 3. Consider the function $f(x, y) = \frac{1}{x^2 + y^2}$. Use a linear approximation to approximate the value of $f(1.1, 1.9)$.

Problem 4. A cylinder has radius 2 and height 3. Suppose that the radius and height each increase by 0.1. Approximately how much will the volume change?