

Math 210 (Lesieutre)
14.2: Line integrals
March 31, 2017

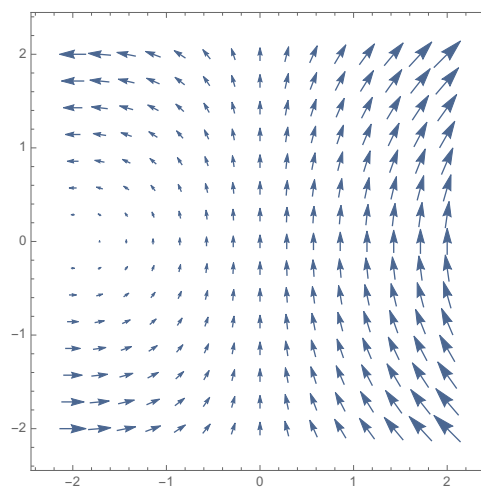
Problem 1. Consider the scalar function $f(x, y) = xy$. Compute $\int_C f ds$ for the listed paths.

a) Let C be a straight line path from $(2, 0)$ to $(0, 2)$.

b) Let C be the part of a circle of radius 2 centered at the origin that lies in the first quadrant, oriented counterclockwise.

c) What is the average value of $f(x, y)$ along the path from (b)?

Problem 2. Now consider the vector field $\mathbf{F}(x, y) = \langle xy, 1 + x \rangle$:



Let C_1 be the quarter-circle path from problem 1b, and C_2 the straight line path from $(0, 0)$ to $(2, 0)$.

a) Do you expect $\int_{C_1} \mathbf{F} \cdot d\mathbf{r}$ to be positive, negative, or zero? How about $\int_{C_2} \mathbf{F} \cdot d\mathbf{r}$?

b) Compute $\int_{C_1} \mathbf{F} \cdot d\mathbf{r}$.

c) Compute $\int_{C_2} \mathbf{F} \cdot d\mathbf{r}$, or at least set up the corresponding single variable integral.

Problem 3. Consider the change of variables T given by $u = 2x + y$ and $v = x + y$. Let R be the parallelogram in the xy -plane with vertices at $(0, 0)$, $(-1, 2)$, $(3, -3)$, and $(2, -1)$. Compute

$$\iint_R xy \, dx \, dy.$$