Math 210 (Lesieutre)
14.3: Conservative vector fields, II
April 5, 2017

Problem 1. Consider the field $\mathbf{F}_1 = \langle x^2, y^2 \rangle$. Last time we saw that this is a conservative field, and that $\phi(x, y) = \frac{x^3}{3} + \frac{y^3}{3}$ is a potential function.

a) Compute the line integral of $\mathbf{F}$ along a straight line path from $(0, 0)$ to $(1, 1)$.

b) Compute the line integral of $\mathbf{F}$ along a path that goes from $(0, 0)$ to $(1, 0)$ and then to $(1, 1)$. What do you notice?

c) Repeat the previous two calculations using the fundamental theorem for line integrals.

Problem 2. Consider the vector field $\mathbf{F}(x, y) = \langle y + 1, x + 1 \rangle$.

a) Verify that $\mathbf{F}$ is a conservative field and find a potential function $\phi(x, y)$.

b) Let C be a semicircular path from $(1, 0)$ to $(-1, 0)$. Use the fundamental theorem for line integrals to compute $\int_C \mathbf{F} \cdot d\mathbf{r}$.

c) Let C be a path that goes the whole way around the unit circle. Check by a direct computation that

$$\int_C \mathbf{F} \cdot d\mathbf{r} = 0.$$

Problem 3. Let $\mathbf{F}$ be the vector field $\langle x, y \rangle$ and let C be a straight line from $(1, 1)$ to $(-1, 1)$.

a) Do you expect the flux of $\mathbf{F}$ across C to be positive, negative, or zero?

b) Compute the flux $\int_C \mathbf{F} \cdot \mathbf{n}\, ds$.