

Math 210 (Lesieutre)
Quiz 10
April 7, 2017

Name: _____

Problem 1. Let $\mathbf{F} = \langle 0, y \rangle$, and let C be a path going around the unit circle counterclockwise, starting at $(1, 0)$. Compute $\oint_C \mathbf{F} \cdot d\mathbf{r}$, using the method of your choice.

This field is conservative! Indeed, $f_y = 0$ and $g_x = 0$. Since it's the integral around a closed loop, the result must be 0.

You could also do this using the definition. The parametrization is $\mathbf{r}(t) = \langle \cos t, \sin t \rangle$, so $\mathbf{r}'(t) = \langle -\sin t, \cos t \rangle$. Plugging into \mathbf{F} , we get $\langle 0, y \rangle = \langle 0, \sin t \rangle$, and the integral becomes

$$\begin{aligned} \oint_C \mathbf{F} \cdot d\mathbf{r} &= \int_0^{2\pi} \langle 0, \sin t \rangle \cdot \langle -\sin t, \cos t \rangle dt = \int_0^{2\pi} \sin t \cos t dt = \int_0^{2\pi} \frac{\sin 2t}{2} dt \\ &= -\frac{\cos 2t}{4} \Big|_0^{2\pi} = 0. \end{aligned}$$

You could also use Green's theorem, if you want. We have $\text{curl } \mathbf{F} = g_x - f_y = 0$, and so

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_R 0 dA = 0.$$