Math 210 (Lesieutre)
11.5: Lines and curves in space
January 20, 2017

Problem 1. *Consider the line from* $(1, 1, 1)$ *to* $(1, 2, 3)$.

a) *Express a parametrization of the line in vector form.*
The direction vector is $\mathbf{v} = (1, 2, 3) - (1, 1, 1) = \langle 0, 1, 2 \rangle$. Then we want the line

$$\mathbf{r}(t) = \langle 1, 1, 1 \rangle + t \langle 0, 1, 2 \rangle .$$

This seems to work: when we plug in $t = 0$, we get $\langle 1, 1, 1 \rangle$, and when we plug in $t = 1$, we get $\langle 1, 2, 3 \rangle$.

b) *Express your parametrization as three functions* $x(t)$, $y(t)$, *and* $z(t)$.
Our parametrization was $\mathbf{r}(t) = \langle 1 + 0t, 1 + 1t, 1 + 2t \rangle$, so we want

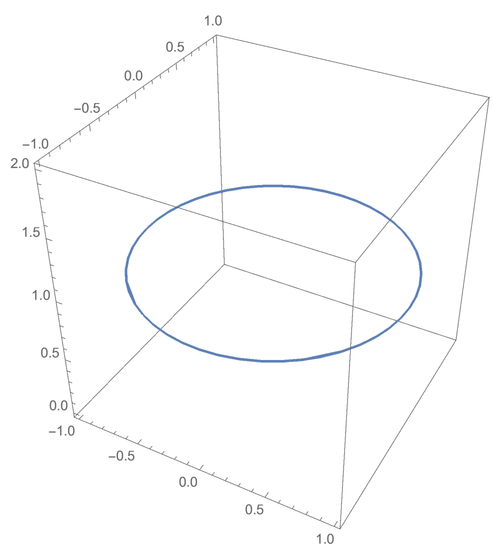
$$x(t) = 1,$$
$$y(t) = 1 + t,$$
$$z(t) = 1 + 2t.$$

c) *At what time* t *does the line cross the plane* $z = 10$?
We have $z(t) = 1 + 2t$, so we just want to know when $1 + 2t = 10$, i.e. $t = 9/2$.

d) *What is the projection of the line onto the* xy*-plane?*
It's $x(t) = 1$ and $y(t) = 1 + t$ (we're in the xy-plane, so it's implicit that $z = 0$ for all t).

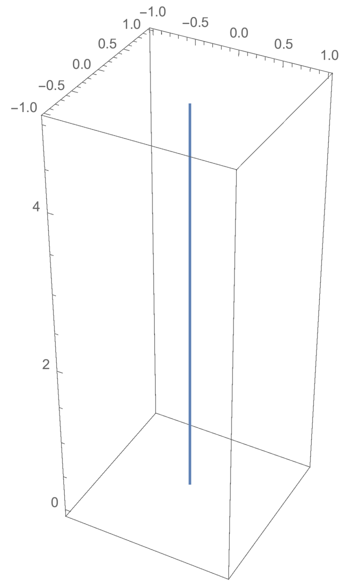
Problem 2. *Sketch the parametrized curves indicated below.*

a) $\mathbf{r}(t) = \langle \cos t, \sin t, 1 \rangle$ *for* $t \leq 0 \leq 4\pi$.
This one is a circle contained in the horizontal plane $z = 1$. For t in the range in question, it goes around the circle twice, in a counterclockwise direction when viewed from above.



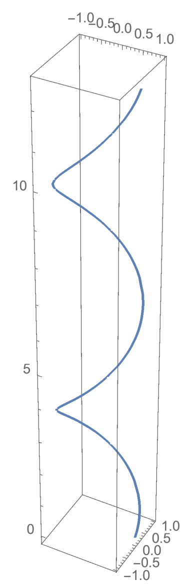
b) $\mathbf{r}(t) = \langle 0, 0, t\rangle$ *for* $0 \leq t \leq 5$

This is just a straight line from $(0, 0, 0)$ moving upwards. It ends at $(0, 0, 5)$.



c) $\mathbf{r}(t) = \langle \cos t, \sin t, t\rangle$ *for* $0 \leq t \leq 4\pi$.

This one is a *helix*. It moves around in a circle, while simultaneously going upwards. Since we go to 4π, it makes two full cycles around.



Problem 3. *Do the two lines*

$$\mathbf{r}_1(s) = \langle s, 1 + s, 2s - 3\rangle\,,$$
$$\mathbf{r}_2(t) = \langle 2 - t, 2t, t\rangle$$

intersect?

To intersect, there must be values of s and t for which $s = 2 - t$, $1 + s = 2t$, and $2s - 3 = t$. This is an overdetermined system: there may or may not be any solutions. The first two are solved by $s = 1$, $t = 1$, but then the third equation isn't satisfied. So there is no s and t that meets both requirements.

Problem 4. *For what value of a do the two lines*

$$\mathbf{r}_1(s) = \langle 3 + s, 1 + s, a + s \rangle \,,$$
$$\mathbf{r}_2(t) = \langle 2t + a, t + a, 0 \rangle$$

intersect?

What does it mean for them to intersect? It means that there is a value s and a value t for which $\mathbf{r}_1(s) = \mathbf{r}_2(t)$ (they might pass through the same point, but at two different values of the parameter).

So we need to solve the equations $3 + s = 2t$, $1 + s = t$, and $a + s = 0$ for s and t, in terms of a. For most values of a there are going to be no solutions, but for some special a there will be one.

First we use the first two to solve for s and t (in principle our answer could involve a, but it's not going to this time). Subtracting the second equation from the first (or doing row reduction, or matrices,. . .), we get $s = 1 + a$ and $t = 2$.

Now we need to know whether the third coordinates are also equal for these values of s and t. Is $a + s = 0$? Well, $s = 1 + a$, so we are asking whether $2a + 1 = 0$. This is the case if $a = -1/2$.

Again: if a has some other value, there is no s and t that make all the components equal. But if $a = -1/2$, the lines do intersect.

Problem 5. *What is* $\lim_{t\to 0^+} \mathbf{v}(t)$*, where* $\mathbf{v}(t) = \left\langle t, \frac{\sin t}{t}, e^{-1/t} \right\rangle$*?*

We just take the limit of each one of the components, whcih gives us $\langle 0, 1, 0 \rangle$.