Math 310, Lesieutre
Midterm #1
September 30, 2015

Name: ______________________________

1. Consider the following system of linear equations:

$$\begin{array}{rcrcrcrcl} x_1 & + & 3x_2 & & & - & 4x_4 & = & 1 \\ 2x_1 & + & 6x_2 & + & x_3 & - & 6x_4 & = & 2 \\ & & & & 2x_3 & + & 4x_4 & = & 0 \end{array}$$

   (a) (15 pts) Find the general solution to this system.

   (b) (10 pts) Express the general solution to the system in parametric vector form.

2. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation defined by

$$T\left(\begin{bmatrix}1\\2\\0\end{bmatrix}\right) = \begin{bmatrix}4\\-4\end{bmatrix}, \quad T\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right) = \begin{bmatrix}1\\-1\end{bmatrix}, \quad T\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right) = \begin{bmatrix}1\\-1\end{bmatrix}$$

   (a) (5 pts) What is

$$T\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right)?$$

   Hint: $\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}1\\2\\0\end{bmatrix} - 2\begin{bmatrix}0\\1\\0\end{bmatrix}$.

   (b) (10 pts) Write down the matrix for the linear transformation T. If you aren't sure about your answer to (a), you can assume $T\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) = \begin{bmatrix}3\\-3\end{bmatrix}$.

   (c) (5 pts) Is this linear transformation onto? Justify. Explain what your answer means about the transformation T.

3. Consider the vectors

$$\mathbf{v}_1 = \begin{bmatrix}1\\-2\\0\end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix}2\\1\\1\end{bmatrix}, \quad \mathbf{v}_3 \begin{bmatrix}6\\-2\\h\end{bmatrix}$$

   (a) (10 pts) For what value(s) of h are the three vectors *linearly dependent*?

   (b) (5 pts) For what value(s) of h is the following matrix *invertible*?

$$\begin{bmatrix}1 & 2 & 6\\-2 & 1 & -2\\0 & 1 & h\end{bmatrix}$$

   Justify your answer. (Hint: the columns of the matrix are the same as the vectors in (a))

4. (a) (10 pts) Set up a system of linear equations that you could use to balance the following chemical reaction, and write down the augmented matrix. You do not need to solve it.

$$Cl_2O + H_3N \longrightarrow NH_4Cl + N_2 + H_20$$

(b) (10 pts) Suppose that 90% of people who are healthy one day are healthy the next day (while the rest become sick), and 60% of people who are sick one day are sick the next day (and the rest are healthy)
On day 0, there are 100 healthy people and 100 sick people. How many healthy and sick people are there on day 2? Compute the answer using a difference equation.

5. Consider the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 7 \end{bmatrix}.$$

(a) (10 pts) Compute the matrix A^2.

(b) (10 pts) Compute the inverse of A using row reduction. (You will only get half credit if you use the formula for inverse of a 2×2 matrix!)