Math 310, Lesieutre
Midterm #2
November 11, 2015

Name: ______________________________

1. Consider the matrix
$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 3 \\ 1 & -1 & 1 \end{bmatrix}$$
   (a) [10] Compute an LU decomposition for A.
   (b) [10] What is the determinant of A?

2. Consider the matrix
$$B = \begin{bmatrix} -1 & 1 \\ 3 & 1 \end{bmatrix}$$
   (a) [12] Find the eigenvalues and eigenvectors of B.
   (b) [8] Give a diagonalization of B (i.e. find P and D so $B = PDP^{-1}$, where D is diagonal).

3. Row reduction on a matrix C yielded the echelon form
$$U = \begin{bmatrix} 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$
   (a) [5] I'm not going to tell you the original matrix C. You still have enough information to find 2 out of the 3 spaces $\operatorname{Row} C$, $\operatorname{Col} C$, and $\operatorname{Nul} C$. Which ones?
   (b) [10] For each of the spaces in your answer to (a), give a basis. What are the dimensions of the three subspaces? (Include the dimension of the one for which you did not find a basis.)
   (c) [10] Let $\mathcal{B}$ be the basis for $\operatorname{Row} C$ that you found in part (b). Another basis $\mathcal{C}$ is given by the two vectors
$$\mathbf{c}_1 = \begin{bmatrix} 1 \\ -2 \\ 3 \\ 3 \end{bmatrix}, \quad \mathbf{c}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}.$$
   Find the change of basis matrix $\underset{\mathcal{B} \leftarrow \mathcal{C}}{\mathcal{P}}$.

4. (a) [10] Use cofactor expansion to compute the determinant of
$$E = \begin{bmatrix} 1 & 1 & 2 \\ -1 & -2 & -2 \\ 2 & 0 & 4 \end{bmatrix}.$$
   (Half credit if you use some other method instead.)

(b) [5] What does your answer to (a) tell you about the columns of E?

5. (a) [10] A Markov process is defined by the stochastic matrix

$$M = \begin{bmatrix} 0.6 & 0.1 \\ 0.4 & 0.9 \end{bmatrix}.$$

What is $\lim_{n\to\infty} M^n \mathbf{x}_0$ if $\mathbf{x}_0 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$? (In other words, what vector does $M^n \mathbf{x}_0$ approach when n is very large?)

(b) [10] Consider the three polynomials

$$\mathbf{p}_1(t) = 1 + t, \quad \mathbf{p}_2(t) = 1 + 2t, \quad \mathbf{p}_3(t) = 2 + 3t.$$

Are these three polynomials linearly independent? Justify your answer. (Hint: think about the coordinate vectors for these polynomials in some basis.)