Announcements.

- I'm gone for the rest of the week; no more office hours.
- Grades on ~~Piazza~~ Blackboard up to date (But it's not dropping the low quizzes)
- Change to schedule: §4.9 next, §5.1 on Monday.

(Re: a question)

Example:

basis: $\mathcal{B}$

$\vec{b}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

these are a basis for $\mathbb{R}^2$

$\vec{b}_2 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

given $\vec{x} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, what is $[\vec{x}]_\mathcal{B}$?

want to get

$$\begin{pmatrix} 3 \\ 4 \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

(solve for c_1, c_2)

find using row reduction: $\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 2 & 3 & 4 \end{array}\right]$ tells you c_1, c_2.

$P_\mathcal{B}$

or $[\vec{x}]_\mathcal{B} = P_\mathcal{B}^{-1}(\vec{x})$.

Change of basis

A given vector space has many possible bases.

Say we have two bases: $\mathcal{B} = \vec{b}_1, \vec{b}_2, \vec{b}_3$

$\mathcal{C} = \vec{c}_1, \vec{c}_2, \vec{c}_3.$

Basic question: given coordinates $[\vec{x}]_{\mathcal{B}}$,

how to find $[\vec{x}]_{\mathcal{C}}$?

First case: two bases for $\mathbb{R}^n$. $\mathcal{B} = \vec{b}_1, \dots, \vec{b}_n$

$[\vec{b}_1 \cdots \vec{b}_n]$ $\quad \mathcal{C} = \vec{c}_1, \dots, \vec{c}_n.$

we know $\vec{x} = P_{\mathcal{B}}([\vec{x}]_{\mathcal{B}})$.

<u>and</u> $\vec{x} = P_{\mathcal{C}}([\vec{x}]_{\mathcal{C}})$.

$$P_{\mathcal{B}}([\vec{x}]_{\mathcal{B}}) = P_{\mathcal{C}}([\vec{x}]_{\mathcal{C}})$$

multiply by $P_{\mathcal{C}}^{-1}$ on left

$$(P_{\mathcal{C}}^{-1} P_{\mathcal{B}})([\vec{x}]_{\mathcal{B}}) = [\vec{x}]_{\mathcal{C}}.$$

great!

to get $[\vec{x}]_{\mathcal{C}}$, just multiply ~~on test~~ by some matrix.

this is called the change of basis matrix, written $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P}$

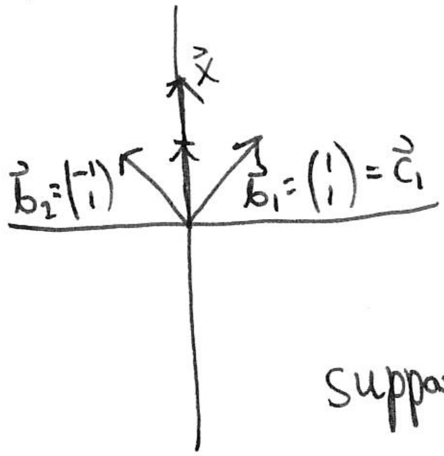
$\mathcal{C} \leftarrow \mathcal{B}$

for two bases in $\mathbb{R}^n$, $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P} = P_{\mathcal{C}}^{-1} P_{\mathcal{B}}$

(a little more complicated for other vector spaces;
if e.g. $P_{\mathcal{B}}$ is ~~change of~~ coord matrix for null(A),
it won't be square, and this doesn't work.)

it will still true that $[\vec{x}]_{\mathcal{C}} = \underset{\mathcal{C}\leftarrow\mathcal{B}}{P}([\vec{x}]_{\mathcal{B}})$, but
how to find $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P}$ is different.

example:



two bases for $\mathbb{R}^2$:

$\mathcal{B}: \binom{1}{1}, \binom{-1}{1}$

$\mathcal{C}: \binom{1}{1}, \binom{0}{1}$

suppose $[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. what's $[\vec{x}]_{\mathcal{C}}$?

$$P_{\mathcal{B}} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$P_{\mathcal{C}} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\underset{\mathcal{C} \leftarrow \mathcal{B}}{P} = P_{\mathcal{C}}^{-1} P_{\mathcal{B}} = \cdots = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$$

$$[\vec{x}]_{\mathcal{C}} = \underset{\mathcal{C} \leftarrow \mathcal{B}}{P} [\vec{x}]_{\mathcal{B}} = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

if $[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\vec{x} = 1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

$[\vec{x}]_{\mathcal{C}} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$, $\vec{x} = 0 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ ✓.