Announements

New office hours (effective immediately!)

Tu 3-4, online ~7-10 (math310john@gmail.com)

Th 3-4

F ~~3-4~~ 10-11

or email me!

- Pick up your last quiz on your way out

  Grades on Blackboard

- Network problem on HW had a backwards arrow! (Not on quiz)

- Quiz!

Linear independence

A set of vectors $\vec{v}_1, \dots, \vec{v}_n$ is <u>linearly independent</u>

if $x_1\vec{v}_1 + x_2\vec{v}_2 + \dots + x_n\vec{v}_n = 0$ has only $x_1 = x_2 = \dots = x_n = 0$.

as a solution.

<u>example</u> (lin. indep.)

$$\vec{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \vec{v}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \vec{v}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

can we have

$$x_1\vec{v}_1 + x_2\vec{v}_2 + x_3\vec{v}_3 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$ but x's aren't all 0?

$$= x_1\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + x_2\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + x_3\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

not equal $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ unless

$x_1 = x_2 = x_3 = 0$.

non-example (not lin. indep)

$\vec{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\vec{v}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $\vec{v}_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

$$(1)\vec{v}_1 + (1)\vec{v}_2 + (-1)\vec{v}_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

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how to tell if $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are linearly independent?

ie. is there a solution to the vector equation

$$x_1\vec{v}_1 + x_2\vec{v}_2 + x_3\vec{v}_3 = \vec{0} \text{ \underline{other than} } \vec{x} = \vec{0}?$$

to check, write down the corresponding matrix eqn

do row reduction, look for free variable.

Check if

$$\begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\2\\3\end{pmatrix}, \begin{pmatrix}1\\3\\5\end{pmatrix} \text{ are independent.}$$

$$\Downarrow$$

$$\begin{pmatrix}1&1&1\\1&2&3\\1&3&5\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix} \text{ have nonzero solution?}$$

$$\left(\begin{array}{ccc|c}1&1&1&0\\1&2&3&0\\1&3&5&0\end{array}\right) \to \left(\begin{array}{ccc|c}1&1&1&0\\0&1&2&0\\1&3&5&0\end{array}\right) \to \left(\begin{array}{ccc|c}1&1&1&0\\0&1&2&0\\0&2&4&0\end{array}\right) \to \left(\begin{array}{ccc|c}1&1&1&0\\0&1&2&0\\0&0&0&0\end{array}\right)$$

$$\downarrow$$

$$\left(\begin{array}{ccc|c}1&0&-1&0\\0&1&2&0\\0&0&0&0\end{array}\right)$$

RREF.

general solution:

$$\begin{cases}x_1 = x_3\\ x_2 = -2x_3\\ x_3 \text{ is free}\end{cases}.$$

this has nonzero solutions!

$x_1 = 1,\ x_2 = -2,\ x_3 = 1$ is a solution

$\Longrightarrow \vec{v}_1, \vec{v}_2, \vec{v}_3$ <u>not</u> linearly independent.

$$(1)\begin{pmatrix}1\\1\\1\end{pmatrix}+(-2)\begin{pmatrix}1\\2\\3\end{pmatrix}+(1)\begin{pmatrix}1\\3\\5\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}.$$

Say there are only 2 vectors,

$\vec{v}_1, \vec{v}_2$. when are they independent?

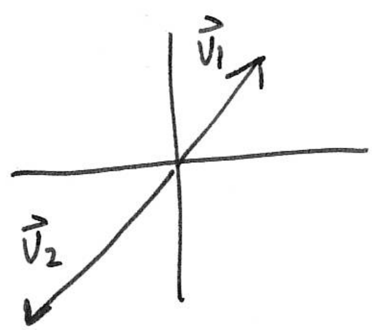
Let's think about when <u>not</u> independent:

this means $a_1\vec{v}_1 + a_2\vec{v}_2 = 0$. where a_1, a_2 are not 0.

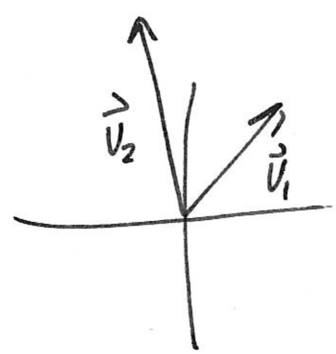
$$\Downarrow$$

$$\vec{v}_1 = -\frac{a_2}{a_1}\vec{v}_2.$$

$\vec{v}_1$ is a multiple of $\vec{v}_2$!



NOT INDEPENDENT.



INDEPENDENT

what about 3 vectors?

not independent:

$$a_1\vec{v}_1 + a_2\vec{v}_2 + a_3\vec{v}_3 = 0,$$ where a's aren't all 0!

$$\Downarrow$$

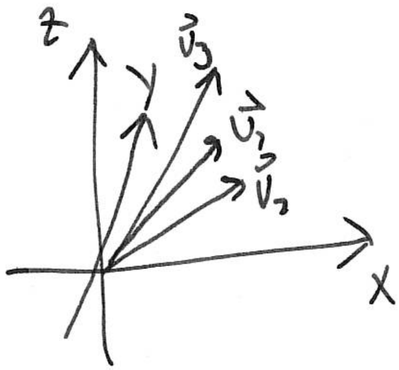
$$\vec{v}_1 = -\frac{a_2}{a_1}\vec{v}_2 - \frac{a_3}{a_1}\vec{v}_3.$$

$\vec{v}_1$ is a linear combo of $\vec{v}_2$ & $\vec{v}_3$!

(assume 3D vectors!)

<u>Geometrically</u>: linear combos of $\vec{v}_2$ and $\vec{v}_3$ are contained in a plane

so $\vec{v}_1$ is too



<u>LINEARLY DEPENDENT:</u>

the three vectors are in a plane

<u>LINEARLY INDEPENDENT</u>

three vectors not in a plane.