Math 310, Lesieutre
Exam 2 Practice
November 11, 2015

Name: ______________________________

This is a version of an old exam replacing some topics we didn't cover yet. Solutions will be posted over the weekend.

1. Consider the matrix $A = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}$.
   (a) What are the eigenvalues of A?
   (b) What are the eigenvectors of A?
   (c) Find a diagonalization of the matrix A.
   (d) Compute the matrix A^5.

2. The matrix B has reduced echelon form U, where

$$B = \begin{bmatrix} 1 & 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & -1 & -1 \\ 1 & 1 & 2 & 1 & 2 \\ 2 & 1 & 3 & -1 & -3 \end{bmatrix}, \qquad U = \begin{bmatrix} 1 & 0 & 1 & 0 & -1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

   (a) Give a basis for Row B. What is the dimension?
   (b) Give a basis for Col B. What is the dimension?
   (c) Give a basis for Nul B. What is the dimension?
   (d) For what values of a and b does the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & a & 2 & 2 \\ 0 & 0 & 0 & b & 2 \end{bmatrix}$$

   have rank 2?

3. Let $\mathcal{E} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ be the standard basis for $\mathbb{R}^2$.
   (a) Show that $\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \end{bmatrix} \right\}$ is another basis for $\mathbb{R}^2$.
   (b) Write down the change of basis matrix $\underset{\mathcal{E}\leftarrow\mathcal{B}}{\mathcal{P}}$ from $\mathcal{B}$ to the standard basis $\mathcal{E}$.
   (c) Find the $\mathcal{B}$-coordinate vector $[\mathbf{v}]_{\mathcal{B}}$ of the vector $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

4. Consider the matrix

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix}.$$

(a) What is the dimension of the eigenspace corresponding to the eigenvalue 1? (You do not need to compute a basis.)

(b) What is the dimension of the eigenspace corresponding to the eigenvalue 2? (You do not need to compute a basis.)

(c) Explain why the matrix C is not diagonalizable.

5. (a) Find an LU decomposition of the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 3 & 7 & 3 & 3 \\ 2 & 2 & 3 & 4 \end{bmatrix}.$$

(b) Use your LU decomposition to find a solution to $A\mathbf{x} = \begin{bmatrix} 1 \\ 4 \\ -1 \end{bmatrix}$.

6. Consider the matrix

$$A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 1 & 1 \\ 0 & 3 & 0 \end{bmatrix}$$

(a) Compute the determinant of A using row reduction.

(b) Compute the determinant of A using cofactor expansion.

(c) Use Cramer's rule to solve

$$A\mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

for x_3. (You do not need to solve for x_1 and x_2.)