Math 310, Lesieutre
Quiz #12
December 2, 2015

Name: ______________________

1. *Consider the system of equations $A\mathbf{x} = \mathbf{b}$, where*

$$A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}.$$

This system is not consistent. Find the least-squares solution $\hat{\mathbf{x}}$.

We need to solve $A^T A\mathbf{x} = A^T\mathbf{b}$.

$$A^T A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & -1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 3 \end{bmatrix}$$

$$A^T\mathbf{b} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Now we just row-reduce.

$$\left[\begin{array}{cc|c} 2 & 2 & 1 \\ 2 & 3 & -1 \end{array}\right] \to \left[\begin{array}{cc|c} 2 & 2 & 1 \\ 0 & 1 & -2 \end{array}\right] \to \left[\begin{array}{cc|c} 2 & 0 & 5 \\ 0 & 1 & -2 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & 5/2 \\ 0 & 1 & -2 \end{array}\right]$$

So the best fit solution is $\hat{\mathbf{x}} = \begin{bmatrix} 5/2 \\ -2 \end{bmatrix}$.

2. *Suppose you want to fit a curve of the form $y = \beta_1 + \beta_2 x + \beta_3 x^3$ through the points $(1, 1)$, $(1, 2)$, $(2, 3)$, and $(-1, 2)$, where β_i are parameters.*
Write down the linear system you would use to find the appropriate values for β_1, β_2, and β_3. (Say what the normal equations are, but you don't need to multiply out all the matrices or solve.)

To go through $(2, 3)$, we need to have $\beta_1 + 2\beta_2 + 8\beta_3 = 3$, and similarly for the other points. This gives the linear system

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 8 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \end{bmatrix}.$$

This has no solutions, so instead we want to solve $A^T A\mathbf{x} = A^T\mathbf{b}$:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & -1 \\ 1 & 1 & 8 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 8 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & -1 \\ 1 & 1 & 8 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \end{bmatrix}$$

3. If you finished early and haven't done your course evaluation yet, please fill it out after you leave!