Math 310, Lesieutre
Quiz #2
September 9, 2015

Name: ____________________

1. *Define*

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} h \\ -1 \\ 1 \end{bmatrix}.$$

For what value(s) of h is $\mathbf{y}$ a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$? (In other words, for what values is it in the plane generated by $\mathbf{v}_1$ and $\mathbf{v}_2$?)

To figure this out, we write down the augmented matrix corresponding to the vector equation and run row reduction.

$$\left[\begin{array}{cc|c} 1 & 2 & h \\ 2 & 5 & -1 \\ -2 & -4 & 1 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 2 & h \\ 0 & 1 & -1-2h \\ -2 & -4 & 1 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 2 & h \\ 0 & 1 & -1-2h \\ 0 & 0 & 1+2h \end{array}\right]$$

This matrix is already in echelon form, so we can tell whether there's going to be a solution or not. The bottom row is a row of the form $[\,0\,0\,|\,b\,]$ with nonzero b (meaning there is no solution) unless $1+2h=0$. Thus if $h=-1/2$, there is a solution, and $\mathbf{y}$ is a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$. For any other h, there are no solutions.

2. *Describe all solutions of $A\mathbf{x} = \mathbf{0}$ in parametric vector form, where*

$$A = \begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{bmatrix}.$$

Lucky for us this matrix is already in reduced echelon form! The variables x_2 and x_4 are free, and the general solution (with parameters s and t for these two variables) is

$$\begin{cases} x_1 = 2s - t, \\ x_2 = s, \\ x_3 = -3t, \\ x_4 = t. \end{cases}$$

In parametric vector form, this solution is

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = s \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ -3 \\ 1 \end{bmatrix}$$

3. *Set up a system of linear equations to balance the following chemical reaction:*

$$\_\_C_4H_{10} + \_\_O_2 \longrightarrow \_\_CO_2 + \_\_H_2O.$$

Write down the augmented matrix corresponding to the system. You do not need to solve it!

Fill in the blanks with x_1, x_2, x_3, and x_4 respectively. Each of the elements C, H, and O gives us a linear equation:

$$\begin{aligned} C:&\quad 4x_1 = x_3 \\ H:&\quad 10x_1 = 2x_4 \\ O:&\quad 2x_2 = 2x_3 + x_4 \end{aligned}$$

The augmented matrix for the system is

$$\left[\begin{array}{rrrr|r} 4 & 0 & -1 & 0 & 0 \\ 10 & 0 & 0 & -2 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array}\right].$$

(You could put the rows in any order; this doesn't make a difference.)