Math 310, Lesieutre
Quiz #6
October 14, 2015

Name: ______________________________

1. *Use Cramer's rule to solve the following system of linear equations, in terms of the parameter s:*

$$(2s)x_1 + x_2 = 3$$
$$4x_1 + (3s)x_2 = 2$$

This is just $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{bmatrix} 2s & 1 \\ 4 & 3s \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}.$$

Cramer's rule tells us that

$$x_1 = \frac{\det A_1(\mathbf{b})}{\det A} = \frac{\begin{vmatrix} 3 & 1 \\ 2 & 3s \end{vmatrix}}{\begin{vmatrix} 2s & 1 \\ 4 & 3s \end{vmatrix}} = \frac{9s-2}{6s^2-4},$$

$$x_2 = \frac{\det A_2(\mathbf{b})}{\det A} = \frac{\begin{vmatrix} 2s & 3 \\ 4 & 2 \end{vmatrix}}{\begin{vmatrix} 2s & 1 \\ 4 & 3s \end{vmatrix}} = \frac{4s-12}{6s^2-4}.$$

One must be a little careful here: if $6s^2 - 4 = 0$, this isn't giving a solution, because the denominator is zero. So we need to add the stipulation that $s \neq \pm\sqrt{2/3}$.

2. *Consider the subset W of $\mathbb{R}^2$ given by all vectors $\begin{bmatrix} x \\ y \end{bmatrix}$ with $-1 \le x \le 1$ and $-1 \le y \le 1$. Is W a subspace? (Hint: find a vector $\mathbf{w}$ in W and a scalar c so that $c\mathbf{w}$ is not in W.)*

Take the vector $\mathbf{w} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. That's in W, since both of the entries are between -1 and 1, as required. Take $c = 2$. Then $c\mathbf{w} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$, which isn't in W. Since we found a vector in W, and a scalar, such that the product isn't in W, we conclude that W isn't a subspace.

3. *Let V be the set of all vectors of the form*

$$\begin{bmatrix} -3a + 2b \\ -a + b \\ a \end{bmatrix}$$

where a and b are scalars. Find two vectors $\mathbf{u}$ and $\mathbf{v}$ such that $V = \operatorname{span}(\mathbf{u}, \mathbf{v})$. Find a matrix A such that V is the column space of A.

Take

$$\mathbf{u} = \begin{bmatrix} -3 \\ -1 \\ 1 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}.$$

Then $V = \operatorname{span}(\mathbf{u}, \mathbf{v})$. To get A, just stick in $\mathbf{u}$ and $\mathbf{v}$ as the columns of A:

$$A = \begin{bmatrix} -3 & 2 \\ -1 & 1 \\ 1 & 0 \end{bmatrix}.$$

The column space of A is the span of its columns, which is the set V we want.