Math 310, Lesieutre
Problem set #10
November 4, 2015

Problems for M 10/26:

5.1.1 Is $\lambda = 2$ an eigenvalue of
$$\begin{bmatrix} 3 & 2 \\ 3 & 8 \end{bmatrix}?$$
Why or why not?

5.1.2 Is $\lambda = -2$ an eigenvalue of
$$\begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}?$$
Why or why not?

5.1.15 Find a basis for the eigenspace corresponding to each listed eigenvalue.
$$A = \begin{bmatrix} 4 & 2 & 3 \\ -1 & 1 & -3 \\ 2 & 4 & 9 \end{bmatrix}, \quad \lambda = 3.$$

5.1.18 Find the eigenvalues of the matrix
$$A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & -3 \end{bmatrix}.$$

5.1.19 For
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix},$$
find one eigenvalue, with no calculation. Justify your answer.

Problems for W 10/28:

5.2.1 Find the characteristic polynomial and the eigenvalues of the matrix
$$A = \begin{bmatrix} 2 & 7 \\ 7 & 2 \end{bmatrix}.$$

5.2.5 Find the characteristic polynomial and the eigenvalues of the matrix
$$A = \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}.$$

5.2.9 Find the characteristic polynomial and and eigenvalues of the matrix. (Hint: it's 3×3, so your best bet is to use cofactor expansion or the special 3×3 formula.

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 3 & -1 \\ 0 & 6 & 0 \end{bmatrix}.$$

5.2.12 Find the characteristic polynomial and and eigenvalues of the matrix (I know this is getting a little repetitive, but this is a very important thing to be able to do!)

$$A = \begin{bmatrix} -1 & 0 & 1 \\ -3 & 4 & 1 \\ 0 & 0 & 2 \end{bmatrix}.$$

5.2.15 Find the eigenvalues, repeated according to their multiplicities (i.e. if an egienvalue is a double root of the characteristic polynomial, list it twice.)

$$A = \begin{bmatrix} 4 & -7 & 0 & 2 \\ 0 & 3 & -4 & 6 \\ 0 & 0 & 3 & -8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problems for F 10/30:

5.3.1 Let $A = PDP^{-1}$, where P and D are the matrices listed below. Compute A^4.

$$P = \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}, \quad D = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}.$$

5.3.3 Let

$$A = \begin{bmatrix} a & 0 \\ 3(a-b) & b \end{bmatrix}.$$

Use the factorization

$$\begin{bmatrix} a & 0 \\ 3(a-b) & b \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$$

to compute A^k, where k is any positive integer (your answer should be in terms of a, b, and k).

5.3.7 Diagonalize the matrix

$$A = \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}.$$

5.3.11 Same deal:

$$A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$$

Hint: the eigenvalues of A are 1, 2, and 3. (But you still have to find the eigenvectors yourself.)