Math 310, Lesieutre
Problem set #13
November 25, 2015

Problems for M 11/16:

6.2.1 Determine whether the following vectors are an orthogonal set:

$$\begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix}, \quad \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 3 \\ -4 \\ -7 \end{bmatrix}.$$

6.2.8 Show that the following vectors are an orthogonal basis for $\mathbb{R}^2$, and express $\mathbf{x}$ as a linear combination of the $\mathbf{u}$'s.

$$\mathbf{u}_1 = \begin{bmatrix} 2 \\ -3 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} 6 \\ 4 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 9 \\ -7 \end{bmatrix}.$$

6.2.10 Show that the following vectors are an orthogonal basis for $\mathbb{R}^3$, and express $\mathbf{x}$ as a linear combination of the $\mathbf{u}$'s.

$$\mathbf{u}_1 = \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}, \quad \mathbf{u}_3 = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix}.$$

6.2.12 Compute the orthogonal projection of $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ onto the line through $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$ and the origin.

6.2.13 Let $\mathbf{y} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} 4 \\ -7 \end{bmatrix}$. Write $\mathbf{y}$ as a sum of two orthogonal vectors, one in he span of $\mathbf{u}$ and one orthogonal to $\mathbf{u}$. (We didn't do one quite like this in lecture; take a look at Example 3 in the book.)

Problems for W 11/20:

6.3.3 Verify that the given vectors are an orthogonal set, and then find the projection of $\mathbf{y}$ onto $W = \text{span}(\mathbf{u}_1, \mathbf{u}_2)$.

$$\mathbf{y} = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}, \quad \mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}.$$

6.3.9 Let W be the subspace spanned by $\mathbf{u}_1$, $\mathbf{u}_2$ and $\mathbf{u}_3$. Write $\mathbf{y}$ as the sum of a vector in W and a vector orthogonal to W.

$$\mathbf{y} = \begin{bmatrix} 4 \\ 3 \\ 3 \\ -1 \end{bmatrix}, \quad \mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} -1 \\ 3 \\ 1 \\ -2 \end{bmatrix}, \quad \mathbf{u}_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix},$$

6.3.12 Find the closest point to $\mathbf{y}$ in the subspace spanned by $\mathbf{v}_1$ and $\mathbf{v}_2$.

$$\mathbf{y} = \begin{bmatrix} 3 \\ -1 \\ 1 \\ 13 \end{bmatrix}, \quad \mathbf{v}_1 = \begin{bmatrix} 1 \\ -2 \\ -1 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -4 \\ 1 \\ 0 \\ 3 \end{bmatrix}.$$

6.3.15 Let

$$\mathbf{y} = \begin{bmatrix} 5 \\ -9 \\ 5 \end{bmatrix}, \quad \mathbf{u}_1 = \begin{bmatrix} -3 \\ -5 \\ 1 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}.$$

Find the distance from $\mathbf{y}$ to the plane in $\mathbb{R}^3$ spanned by $\mathbf{u}_1$ and $\mathbf{u}_2$. (Hint: what point in that plane is closest to $\mathbf{y}$?)

Problems for F 11/22:

6.4.1 The given set is a basis for a subspace W. Use the Gram–Schmidt process to produce an orthogonal basis for W.

$$\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}, \quad \begin{bmatrix} 8 \\ 5 \\ -6 \end{bmatrix}.$$

6.4.9 Find an orthogonal basis for the column space of the matrix

$$A = \begin{bmatrix} 3 & -5 & 1 \\ 1 & 1 & 1 \\ -1 & 5 & -2 \\ 3 & -7 & 8 \end{bmatrix}.$$

6.4.10 Find an orthogonal basis for the column space of the matrix below.

$$A = \begin{bmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix}$$

6.4.13 The columns of Q below were obtained by running Gram–Schmidt orthonormalization on the columns of A. Find an upper-triangular R with $A = QR$.

$$A = \begin{bmatrix} 5 & 9 \\ 1 & 7 \\ -3 & -5 \\ 1 & 5 \end{bmatrix}, \quad Q = \begin{bmatrix} 5/6 & -1/6 \\ 1/6 & 5/6 \\ -3/6 & 1/6 \\ 1/6 & 3/6 \end{bmatrix}$$