Math 310, Lesieutre
Problem set #14
November 25, 2015

Problems for M 11/23:

6.5.9 Find (a) the orthogonal projection of $\mathbf{b}$ onto $\operatorname{Col} A$ and (b) a least-squares solution to $A\mathbf{x} = \mathbf{b}$.

$$A = \begin{bmatrix} 1 & 5 \\ 3 & 1 \\ -2 & 4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 4 \\ -2 \\ -3 \end{bmatrix}.$$

6.5.12 Same deal, with

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \\ -1 & 1 & -1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ 5 \\ 6 \\ 6 \end{bmatrix}.$$

6.6.2 Find the equation of the least-squares line that best fits the points $(1, 0), (2, 1), (4, 2), (5, 3)$.

6.6.3 Find the equation of the least-squares line that best fits the points $(-1, 0), (0, 1), (1, 2), (2, 4)$.

Problems for W 11/25:

6.5.3 Find a least-squares solution of $A\mathbf{x} = \mathbf{b}$ by (a) constructing the normal equations for $\hat{\mathbf{x}}$ and (b) solving for $\hat{\mathbf{x}}$. ("Normal equations" is just the lingo for $A^T A\mathbf{x} = A^T\mathbf{b}$).

$$A = \begin{bmatrix} 1 & -2 \\ -1 & 2 \\ 0 & 3 \\ 2 & 5 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ 1 \\ -4 \\ 2 \end{bmatrix}.$$

6.5.15 Use the factorization $A = QR$ to find the least-squares solution of $A\mathbf{x} = \mathbf{b}$.

$$A = \begin{bmatrix} 2 & 3 \\ 2 & 4 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2/3 & -1/3 \\ 2/3 & 2/3 \\ 1/3 & -2/3 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 7 \\ 3 \\ 1 \end{bmatrix}.$$

6.6.8 A simple curve that often makes a good model for the variable costs of a company, as a function of the sales level x, has the form $y = \beta_1 x + \beta_2 x^2 + \beta_3 x^3$. There is no constant term because fixed costs are not included.

Give the design matrix and the parameter vector for the linear model that leads to a least-squares fit of the equation above, with data $(x_1, y_1), \ldots, (x_n, y_n)$. (Your answer should be in terms of the x's and y's.)

6.6.9 A certain experiment produces the data $(1, 7.9)$, $(2, 5.4)$, $(3, -0.9)$. Describe the model that produces a least-squares fit of these points by a function of the form $y = A\cos x + B\sin x$. (It'll be a mess to actually find A and B; just set up the matrices.)