Math 310, Lesieutre
Problem set #2
September 9, 2015

Problems for M 8/31:

1.3.11 Determine if $\mathbf{b}$ is a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$, where

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \quad \mathbf{a}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \quad \mathbf{a}_3 = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}.$$

1.3.18 Define

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -3 \\ 1 \\ 8 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} h \\ -5 \\ -3 \end{bmatrix}.$$

For what values of h is $\mathbf{y}$ in the plane generated by $\mathbf{v}_1$ and $\mathbf{v}_2$?

Hint: $\mathbf{y}$ is in the plane if $\mathbf{y}$ is a linear combination of $\mathbf{b}_1$ and $\mathbf{b}_2$. Figure out which values of h will give a consistent linear system by running row reduction in terms of h, and seeing which values of h leave you with a row $[0\,0\,|\,b]$ at the end.

1.4.1 Compute the product $A\mathbf{x}$ using (a) the definition and (b) the row-vector method. If the product is undefined, say why.

$$\begin{bmatrix} -4 & 2 \\ 1 & 6 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 7 \end{bmatrix}$$

1.4.3 Compute the product $A\mathbf{x}$ using (a) the definition and (b) the row-vector method. If the product is undefined, say why.

$$\begin{bmatrix} 6 & 5 \\ -4 & -3 \\ 7 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

1.4.10 Write the system first as a vector equation and then as a matrix equation.

$$\begin{aligned} 8x_1 - x_2 &= 4 \\ 5x_1 + 4x_2 &= 1 \\ x_1 - 3x_2 &= 2 \end{aligned}$$

Problems for W 9/2:

1.5.5 Write the solution set of the given homogeneous system in parametric vector form.

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 0 \\ -4x_1 - 9x_2 + 2x_3 &= 0 \\ -3x_2 - 6x_3 &= 0 \end{aligned}$$

1.5.7 Describe all solutions of $A\mathbf{x} = \mathbf{0}$ in parametric vector form, where A is row equivalent to the given matrix.

$$A = \begin{bmatrix} 1 & 3 & -3 & 7 \\ 0 & 1 & -4 & 5 \end{bmatrix}.$$

1.5.9 Same as above, but with

$$A = \begin{bmatrix} 3 & -9 & 6 \\ -1 & 3 & -2 \end{bmatrix}.$$

1.5.15 Follow the method of Example 3 to describe the solutions of the following system in parametric vector form. Also give a geometric description of the solutions set and compare it to that in Exercise 1.5.5.

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 1 \\ -4x_1 - 9x_2 + 2x_3 &= -1 \\ -3x_2 - 6x_3 &= -3 \end{aligned}$$

Problems for F 9/4:

1.6.1 Suppose an economy has only two sectors, Goods and Services. Each year, Goods sells 80% of its output to Services and keeps the rest, while Services sells 70% of its output to Goods and retains the rest. Find the equilibrium prices for the annual outputs of the Goods and Services sectors that make each sector's income match its expenses.

1.6.5 Balance the unbalanced chemical equation:

$$B_2S_3 + H_2O \to H_3BO_3 + H_2S.$$

1.6.6 Balance the unbalanced chemical equation:

$$Na_3PO_4 + Ba(NO_3)_2 \to Ba_3(PO_4)_2 + NaNO_3.$$

1.6.11 Find the general flow pattern of the network shown in the figure. Assuming that the flows are all nonnegative, what is the largest possible value for x_3?