Math 310, Lesieutre
Problem set #3
September 16, 2015

Problems for W 9/9:

1.7.1 Determine if the following vectors are linearly independent:

$$\begin{bmatrix}5\\0\\0\end{bmatrix}, \quad \begin{bmatrix}7\\2\\-6\end{bmatrix}, \quad \begin{bmatrix}9\\4\\-8\end{bmatrix}.$$

1.7.4 Determine if the following vectors are linearly independent:

$$\begin{bmatrix}-1\\4\end{bmatrix}, \quad \begin{bmatrix}-2\\-8\end{bmatrix}.$$

1.7.9 For what values of h is $\mathbf{v}_3$ in the span of $\mathbf{v}_1$ and $\mathbf{v}_2$? Also, for what values of h is $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ linearly *dependent*? (Think about why this happens!)

$$\begin{bmatrix}1\\-3\\2\end{bmatrix}, \quad \begin{bmatrix}-3\\9\\-6\end{bmatrix}, \quad \begin{bmatrix}5\\-7\\h\end{bmatrix}.$$

1.7.29 Find examples of 3×2 matrices A and B such that $A\mathbf{x} = \mathbf{0}$ has only the trivial solution, and such that $B\mathbf{x} = \mathbf{0}$ has a nontrivial solution.

Problems for F 9/11:

1.8.1 Let

$$A = \begin{bmatrix}2 & 0\\0 & 2\end{bmatrix},$$

and define $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $T(\mathbf{x}) = A\mathbf{x}$. Find the images under T of

$$\mathbf{u} = \begin{pmatrix}1\\-3\end{pmatrix} \text{ and } \mathbf{v} = \begin{pmatrix}a\\b\end{pmatrix}.$$

1.8.3 Let T be defined by $T(\mathbf{x}) = A\mathbf{x}$, where

$$A = \begin{pmatrix}1 & 0 & -2\\-2 & 1 & 7\\3 & -2 & -5\end{pmatrix}.$$

Find a vector $\mathbf{x}$ whose image under T is $\mathbf{b} = \begin{pmatrix}-1\\7\\-3\end{pmatrix}$, and determine whether $\mathbf{x}$ is unique.

1.8.15 Define

$$T(\mathbf{x}) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}.$$

Use a rectangular coordinate system to plot $\mathbf{u} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -2 \\ -4 \end{pmatrix}$ and their images under T. Describe geometrically what T does to each vector $\mathbf{x}$ in $\mathbb{R}^2$.

1.8.16 Same deal as the previous one, but with

$$T(\mathbf{x}) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}..$$