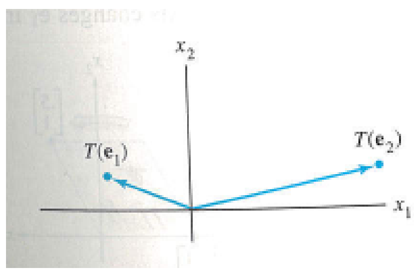
Math 310, Lesieutre
Problem set #4
September 23, 2015

Problems for M 9/14:

1.9.2 Suppose we have a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$, with $T(\mathbf{e}_1) = (1,3)$, $T(\mathbf{e}_2) = (4,-7)$, and $T(\mathbf{e}_3) = (-5,4)$ (where $\mathbf{e}_1 = (1,0,0)$, $\mathbf{e}_2 = (0,1,0)$, and $\mathbf{e}_3 = (0,0,1)$). Find the standard matrix for T. (Note: I originally typed the problem for 1.9.1 instead; if you already did that one, that's OK.)

1.9.4 Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the transformation that rotates points (about the origin) through $-\pi/4$ radians (i.e. $\pi/4$ radians, clockwise). Hint: $T(\mathbf{e}_1) = (1/\sqrt{2}, -1/\sqrt{2})$.

1.9.13 Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation such that $T(\mathbf{e}_1)$ and $T(\mathbf{e}_2)$ are the vectors shown in the figure. Using the figure, sketch the vector $T(2,1)$.



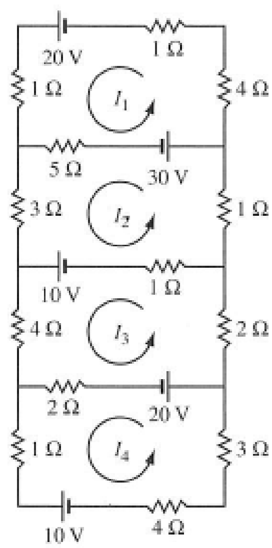
1.9.19 Show that the transformation T below is a linear transformatino by finding a matrix that implements the mapping. Note that $x_1, x_2, \ldots$ are not vectors but are entries in vectors.

$$T(x_1, x_2, x_3) = (x_1 - 5x_2 + 4x_3, x_2 - 6x_3).$$

Problems for W 9/16:

1.9.26 Determine whether the linear transformation T from Exercise 1.9.2 is (a) one-to-one and (b) onto.

1.10.5 Write a matrix that determines the loop currents in the depicted circuit. You don't need to solve for the loop currents.

5. 

1.10.9 In a certain region, about 7% of a city's population moves to the usrrounding suburbs each year, and about 5% of the suburban population moves into the city. In 2015, there were 800,000 residents in the city and 500,000 in the suburbs. Set up a difference equation that describes this situation, where $\mathbf{x}_0$ is the initial population in 2015. Then estimate the populations in the city and in the suburbs two years later, in 2017.

Problems for F 9/18:

2.1.1 Consider the matrices

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix}$$
$$C = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 3 & 5 \\ -1 & 4 \end{bmatrix}.$$

Compute $-2A$, $B - 2A$, AC, and CD.

2.1.3 Let $A = \begin{bmatrix} 4 & -1 \\ 5 & -2 \end{bmatrix}$. Compute $3I_2 - A$ and $(3I_2)A$.

2.1.5 Compute the product AB in two ways: (a) using the definition, where $A\mathbf{b}_1$ and $A\mathbf{b}_2$ are computed separately (b) using the row–column rule for computing AB. Here

$$A = \begin{bmatrix} -1 & 2 \\ 5 & 4 \\ 2 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix}.$$

2.1.10 Let $A = \begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 8 & 4 \\ 5 & 5 \end{bmatrix}$, $C = \begin{bmatrix} 5 & -2 \\ 3 & 1 \end{bmatrix}$. Verify that $AB = AC$,even though $B \neq C$.