Math 553, Lesieutre
Problem set #4
due February 12, 2016

1. Give an example of a scheme that is:
   (a) Noetherian, but not of finite type over any field (i.e. does not admit any morphism to $\operatorname{Spec} k$ for any k)
   (b) Connected, and of locally finite type over $\operatorname{Spec} \mathbb{C}$, but not of finite type over $\operatorname{Spec} \mathbb{C}$.
   (c) Neither reduced nor irreducible.
   (d) Not reduced, but has a dense open subscheme which is reduced.
2. Let $Y = \operatorname{Spec} \mathbb{C}[x, y]/(y - x^3 - x)$ and $Z = \mathbb{C}[x, y]/(x, y)^2$ be two closed subschemes of $\operatorname{Spec} \mathbb{C}[x, y]$. Describe the scheme-theoretic intersection $Y \cap Z$ inside X.
3. Find an example of a scheme X over $\operatorname{Spec} \mathbb{R}$ which is irreducible, but such that $X \times_{\mathbb{R}} \operatorname{Spec} \mathbb{C}$ is not irreducible. Describe the fibers of the map $X \times_{\mathbb{R}} \operatorname{Spec} \mathbb{C} \to X$. (A scheme over $\operatorname{Spec} k$ is called *geometrically irreducible* if $X \times_{\operatorname{Spec} k} \operatorname{Spec} \bar{k}$ is irreducible; you've found a scheme that isn't.)
4. Consider the family $\operatorname{Spec} \mathbb{C}[x, y, t]/(ty^2 + (1 - t)(x^2 - y^2)) \to \operatorname{Spec} \mathbb{C}[t]$. Which of the fibers over closed points are irreducible? Which of the fibers are reduced?
5. II.3.6
6. II.3.8
7. II.4.1
8. II.4.3