18.02 Recitation
Problems
12 September 2011

1. Compute the following dot products. Before you actually work it out, try to estimate the answer by visualizing the vectors.

   (a) $\langle -1, 2\rangle \cdot \langle 1, 2\rangle$

   (b) $\langle 3, 4, 5\rangle \cdot \langle -4, 3, 8\rangle$

   (c) $\langle 3, 4\rangle \cdot \langle 1, -1\rangle$

   (d) $\langle 2, 2, 0\rangle \cdot \langle 0, 0, 1\rangle$

2. Let $\vec{v} = \langle 2, -2, 1\rangle$. What is the component of $\vec{v}$ in the direction of the vector $(3, 2, -6)$?

3. Let ℓ be a line through $(0, 0)$ and $(2, 3)$. What is the distance from ℓ to the point $(6, 0)$? Can you find the closest point on the line?

4. (1A-12) Label the four vertices of a parallelogram in counterclockwise order as $OPQR$. Prove that the line segment from O to the midpoint of $\overrightarrow{PQ}$ intersects the diagonal $\overrightarrow{PR}$ in a point X that is $1/3$ of the way from P to R.

5. (1B-13) Prove that $\cos(\theta_1 - \theta_2) = \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2$.

6. (1C-3). Find the area of the plane triangle whose vertices lie at:

   (a) $(0, 0), (1, 2), (1, -1)$

   (b) $(1, 2), (1, -1), (2, 3)$

7. (1B-6) Let O be the origin, c a given number, and $\hat{u}$ a given direction (i.e. a unit vector). Describe geometrically the locus of points P in space such that $\overrightarrow{OP} \cdot \hat{u} = c\left|\overrightarrow{OP}\right|$. What about the set of points P in space such that $\overrightarrow{OP} \cdot \hat{u} = c$?

8. (1B-9) Let $\vec{A}$ and $\vec{B}$ be two plane vectors, neither one of which is a multiple of the other. Express $\vec{B}$ as the sum of two vectors, one a multiple of $\vec{A}$ and the other perpendicular to $\vec{A}$; give the answer in terms of $\vec{A}$ and $\vec{B}$.

9. Prove the Cauchy-Schwarz inequality: $|\vec{a} \cdot \vec{b}| \leq |\vec{a}||\vec{b}|$. Deduce from this the triangle inequality $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$.

10. For some value of t, the four points $(1, 0, 0)$, $(-1/2, \sqrt{3}/2, 0)$, $(-1/2, -\sqrt{3}/2, 0)$, and $(0, 0, t)$ form a regular tetrahedron. What is the value of t? For some other value of t these form four vertices of a regular dodecahedron – what is this value?