

18.02 Recitation  
Problems  
12 October 2011

1. *Think back to the topographic map from last week. In what direction does the gradient point at the spot indicated below?*

Don't have a file for the map I used, sorry! But it's orthogonal to the level curves, which allows you to sketch it. Remember also that it points in the direction of fastest increase of  $h$ , which on a topographic map means directly uphill.

2. *Suppose you hike northeast. Estimate the gradient and the directional derivative  $D_{\langle 1,1 \rangle} h$ .*

The idea is to count how many level curves you would cross if you went one mile to the east. Multiply that by 20 feet, and you get the number of feet you went up per one mile of horizontal movement.

3. Consider the surface defined by  $x^4 + y^4 + z^4 = 18$ . What is the tangent plane to this surface as  $(2, 1, 1)$ ?

Well, this is a level curve of the function  $f(x, y, z) = x^4 + y^4 + z^4$ , and so the tangent plane is normal to the gradient of the function. The gradient is  $\nabla f = \langle 4x^3, 4y^3, 4z^3 \rangle$ , which at this point is  $\langle 32, 4, 4 \rangle$ . The equation for a plane normal to  $\langle 32, 4, 4 \rangle$  and passing through the point  $(2, 1, 1)$  is

$$32(x - 2) + 4(y - 1) + 4(z - 1) = 0.$$

4. *(2D-5) The temperature at  $(x, y, z)$  is given by  $f(x, y, z) = x^2 + 2y^2 + 2z^2$ . Suppose you are at  $(1, 1, 1)$ . In what direction should you go to get the most rapid decrease in  $f$ ? How far should you go in this direction to get a decrease of 1.2? In what directions is the temperature constant? How far should you go in the direction  $\langle 1, -2, 2 \rangle$  to get an increase of 0.10?*
5. *(2I-2) What point  $P$  on the in the first quadrant and on the surface  $x^3y^2z$  is closest to the origin?*
6. *(2I-4) In an open-top wooden drawer, the two sides and back cost 2/sq. ft., the bottom 1/sq. ft. and the front 4/sq.ft. Using Lagrange multipliers, show that the following problems lead to the same set of three equations in , plus a different fourth equation, and they have the same solution.*

(a) *Find the dimensions of the drawer with largest capacity that can be made for a total wood cost of 72.*

(b) *Find the dimensions of the most economical drawer having volume 24 cu. ft.*

Solutions to these are written up in the course notes!