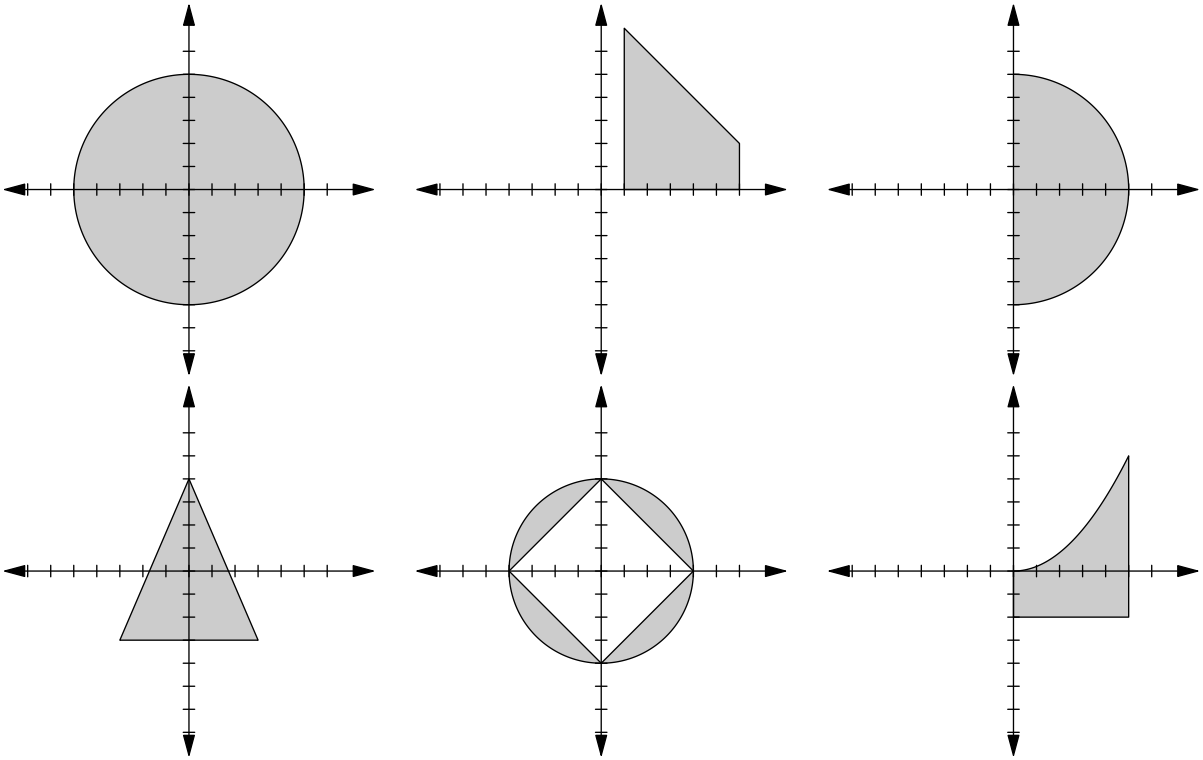
18.02 Recitation
Problems
17 October 2011

1. Compute the integral of the function $f(x, y) = xe^{xy}$ over the rectangle $-2 \leq x \leq 2$, $-1 \leq y \leq 1$. Do you get the same answer for both orders of integration?

2. What about the same function, integrated over the region between the above rectangle and the rectangle $-4 \leq x \leq 3$, $-2 \leq y \leq 3$?

3. Integrate the function $f(x, y) = 1$ over the unit circle. Does your answer make sense? Can you do the integral in polar coordinates?

4. Set up limits of integration for integrals of $f(x, y)$ over the following regions. Can you find more than one way? In particular, try to use polar coordinates for the first, third, and fifth examples.



5. Evaluate the integral

$$\int_{x=1}^{2} \int_{y=0}^{x} \frac{1}{(x^2 + y^2)^{3/2}} \, dy \, dx$$

by converting to polar coordinates.

6. How do the bounds of integration change when you translate a region in the plane by (a, b)? When you rotate the region clockwise by an angle ψ? Rescale by a factor λ?

7. Find the volume of a circular cone with base a circle of radius r and vertex at $(0, 0, h)$.

8. Soon we'll talk about integrals in three dimensional space, where the volume element is $dx\, dy\, dz$. Can you figure out how to express this element in cylindrical coordinates (r, θ, h)? Spherical coordinates?