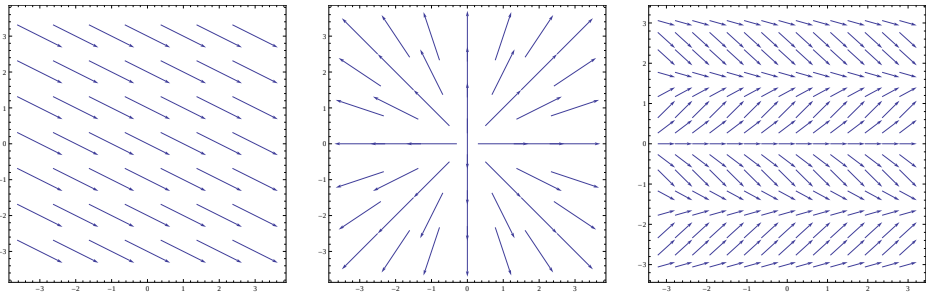
18.02 Recitation
Problems
31 October 2011

1. Sketch the following vector fields:
   (a) $\vec{F}_1(x, y) = y\hat{\imath} + \hat{\jmath}$
   (b) $\vec{F}_2(x, y) = y \cos x\, \hat{\imath} + \sin x\, \hat{\jmath}$
   (c) $\vec{F}_3(x, y) = -\frac{y}{\sqrt{x^2+y^2}}\hat{\imath} + \frac{x}{\sqrt{x^2+y^2}}\hat{\jmath}$

2. Give the equation defining each of the vector fields illustrated below:



3. (4B-2) Evaluate the integral of $\vec{F} = x\hat{\imath} + y\hat{\jmath}$ along the path C which goes once counter-clockwise around a circle of radius r centered at the origin. First argue geometrically, and then check your answer by directly computing the integral.

4. (4B-3) Let $\vec{F} = \hat{\imath} + \hat{\jmath}$. How would you place a directed line segment C of length one so that the value of $\int_C \vec{F} \cdot d\vec{r}$ would be a) a maximum; b) a minimum; c) zero; d) what would the maximum and minimum values of the integral be?

5. For the first and second vector fields of problem 1, compute the integral along three different paths from $(0, 0)$ to $(1, 1)$:
   (a) C_1, a straight line from $(0, 0)$ to $(1, 1)$
   (b) C_2, a line from $(0, 0)$ to $(1, 0)$ and then to $(1, 1)$.
   (c) C_3, along the parabola $y = x^2$

6. One of the vector fields from the previous question is conservative. Which one? Find a function $f(x, y)$ of which it is the gradient, and evaluate the above integral using the fundamental theorem for line integrals.

7. What is the gradient field associated with the function $f(r, \theta) = r \log r$?