18.02 Recitation
Problems
2 November 2011

1. Integrate the field $\vec{F}(x, y) = \hat{\imath}$ over a circular path from $(1, 0)$ to $(0, 1)$. First do this by parametrizing the path, and then use the fact that it is a gradient field and apply the fundamental theorem of calculus for line integrals.

2. Compute
$$\int_C (4x^3 + 6xy^2)\,dx + (6x^2y + 8y^3)\,dy$$
where C is a straight line from $(0, 1)$ to $(1, 0)$ using the definition of a line integral.

3. Two of the following four fields are gradient fields. Figure out which are, and for each, give the corresponding potential function.
   (a) $\vec{F} = \langle 4x^3 + 6xy^2, 6x^2y + 8y^3 \rangle$
   (b) $\vec{F} = \langle x^2 \sin y, -xy \sin y \rangle$
   (c) $\vec{F} = \langle ye^y - y\sin(xy), xe^y - x\sin(xy) \rangle$
   (d) $\vec{F} = \langle 6xy^3 + 1, 9x^2y^2 \rangle$

4. Double-check your answer to problem 2 by using the fundamental theorem.

5. What are the maximum and minimum values that can be achieved by an integral
$$\int_C (4x^3 + 6xy^2)\,dx + (6x^2y + 8y^3)\,dy,$$
where C is a path contained inside the unit circle?

6. Find streamlines of the vector field $\vec{F}(x, y) = \langle -y, x \rangle$.

7. (4C-5a) For what value of a is the field $\vec{F} = (y^2 + 2x)\,\hat{\imath} + axy\,\hat{\jmath}$ a gradient field? What is the potential function for this value of a?