18.02 Recitation
Problems
7 November 2011

1. (4F1) Compute div F and curl F for the fields $\vec{F}_1(x,y) = a\,\hat{\imath} + b\hat{\jmath}$, $\vec{F}_2(x,y) = x^2\,\hat{\imath} + y^2\,\hat{\jmath}$, $\vec{F}_3(x,y) = xy(\hat{\imath} + \hat{\jmath})$.

2. (4D1a) Let $\vec{F}(x,y)$ be the vector field $2y\,\hat{\imath} + x\hat{\jmath}$ and C the unit circle (oriented clockwise). Evaluate $\int_C \vec{F}\,d\vec{r}$ both directly, using the definition of a line integral, and using Green's theorem to find an equivalent double integral.

3. Use Green's theorem to compute $\oint_C (y^2 + y)dx - 2xy\,dy$ where C follows the curve $y = 1 - x^2$ from $(1,0)$ to $(-1,0)$ and then the x-axis, from $(-1,0)$ back to $(1,0)$.

4. Find the area of the ellipse $x^2/a^2 + y^2/^2 = 1$, using the identity
$$\oint x\,dy = \iint_R 1\,dA.$$

5. Consider the vector field $\vec{F}(x,y) = \hat{\imath}$ on the unit circle. Parametrize the circle by $\vec{r}(t) = (\cos t, \sin t)$. For which values of t is $\vec{F} \cdot \vec{n}$ positive? For which is it negative? Evaluate $\int_C \vec{F} \cdot \vec{n}\,ds$ using Green's theorem, and explain why the answer makes sense.

6. (4F3) Verify Green's theorem in the normal form by calculating both sides and showing they are equal if $\vec{F} = x\,\hat{\imath} + y\,\hat{\jmath}$, and C is formed by the upper half of the unit circle and the x-axis interval $[1,1]$.

7. Show that
$$\int_C -\frac{y}{x^2+y^2}\,dx + \frac{x}{x^2+y^2}\,dy = 2\pi$$
For any closed curve C that contains the origin. Note: the field isn't continuous at $(0,0)$, so you can't just integrate over the interior of C!