18.02 Recitation
Problems
9 November 2011

1. Consider the field $\vec{F} = \hat{\imath} + \hat{\jmath}$. How should a line segment be oriented so that the flux of $\vec{F}$ across the segment is maximized? Minimized? Zero?

2. (4F3) Verify Green's theorem in the normal form by calculating both sides and showing they are equal if $\vec{F} = x\,\hat{\imath} + y\,\hat{\jmath}$, and C is formed by the upper half of the unit circle and the x-axis interval $[1, 1]$.

3. Compute the area of a triangle with vertices at $(0,0)$, $(1,0)$, and $(0,1)$ in the hardest possible way: use Green's theorem to convert the double integral for area into a line integral and evaluate.

4. Which of these regions are simply connected?
   (a) A star-shaped set
   (b) The unit circle
   (c) The unit disk
   (d) The right half-plane
   (e) An annulus

5. We've seen that

$$\int_C -\frac{y}{x^2+y^2}\,dx + \frac{x}{x^2+y^2}\,dy = 2\pi$$

   where C is a circle around the origin. What is the integral over a path that goes twice around the origin?

6. Think about the integral

$$\int_C \left(-\frac{y}{x^2+y^2} - \sqrt{2}\frac{y}{(x-2)^2+y^2}\right) dx + \left(\frac{x}{x^2+y^2} + \sqrt{2}\frac{x-2}{(x-2)^2+y^2}\right) dy.$$

   What is its integral around a circle of radius 1 centered at $(0,0)$? At $(2,0)$? Describe the integral around some other paths of your choosing.

7. Set up the triple integral in rectangular coordinates to compute the center of mass of a hemisphere. What are the bounds? If you can, evaluate the integral (this will be hard – later we will study spherical coordinates, in which it's more tractable)!