18.02 Recitation
Problems
28 November 2011

1. Consider the field $\vec{F} = \langle x^2, 2y, z\rangle$. Compute the flux across a square in space with vertices at $(\pm 1, \pm 1, 1)$ by integrating directly. How would the answer change if we changed the function for the first entry of $\vec{F}$?

2. Find the flux of $\vec{F} = \langle x, y, z\rangle$ across the unit sphere via direct computation, and without any computations.

3. Find the flux of $\vec{F} = \langle x^2 z, 2y, z\rangle$ across a cylinder whose base is a circle in the xy-plane of radius 3 and whole height is 2. Include both the side of the cylinder and its top and bottom faces.

4. Redo for the computation for the cylinder above using the divergence theorem. The divergence is $2xz + 2 + 1 = 2xz + 3$.

5. Find the flux of $\vec{F} = \langle x, y, z\rangle$ across the paraboloid $z = x^2 + y^2$ in the region over the square with vertices $(\pm 1, \pm 1)$. Interpret the sign of your answer.

6. Parametrize the following surfaces:

   (a) The front face of a $2 \times 2 \times 2$ cube centered at the origin.

   (b) The part of the unit sphere in the first octant.

   (c) The face of a triangle with vertices at $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$.

   (d) A the paraboloid $z = 1 - x^2 - y^2$ in the region where z is positive.

   (e) A cylinder of radius 2 around the z-axis, bounded by $z = 0$ and $z = 3$.