

## Exam I review problems

Solutions posted at <http://math.mit.edu/~john1/1803/>

1. (1.3.6) Consider the differential equation  $\frac{dy}{dx} = x - y + 1$ . Sketch some isoclines and describe the behavior of the integral curves. Which solutions have local maxima or minima? Points of inflection? Can you identify any fences?
2. (1.4.17) Consider the equation  $y' = 1 + x + y + xy$ . Find the general solution in two different ways: first by observing that it's separable, and then by putting it in reduced standard linear form and applying our general method.
3. (1.5.13) Solve  $y' + y = e^x$ , subject to  $y(0) = 1$ .
4. Consider the logistic equation with stocking/harvesting

$$\frac{dy}{dt} = y(2 - y) + a.$$

What are the equilibria, in terms of  $a$ ? Draw the bifurcation diagram. For what value of  $a$  does there exist a semistable equilibrium? Draw the phase line for several values of  $a$ .

5. (a) Use basic facts about complex numbers to prove the Pythagorean identity  $\cos^2 \theta + \sin^2 \theta = 1$ .  
(b) Express  $\tan \theta$  in terms of  $e^{i\theta}$ .  
(c) How many solutions are there to  $z^3 = i$ ? Compute them.
6. Use Euler's method to approximate  $y(1)$  for a solution to  $y' = y$  with  $y(0) = 1$ . What is the true value? What estimate do you get for the final value when your step size is  $1/n$ , and what happens when  $n \rightarrow \infty$ ?
7. (a) Sketch the graph of  $f(t) = 3 \cos(2t - \pi)$ .  
(b) Express  $2 \cos(2t) + 3 \sin(2t)$  in standard form.  
(c) What is  $\int e^t \sin 2t dt$ ?