Exam III review

1. Let $f(t)$ be the function of period 2 with $f(t) = 1$ for $0 \le t < 1$ and $f(t) = 0$ for $1 \le t < 2$.

   (a) Compute the Fourier series for f.

   (b) How would you solve the differential equation $x' + 3x = f(t)$ using Fourier series?

2. Consider the function
$$f(t) = \begin{cases} 0 & \text{for } t < 0, \\ t^2 & \text{for } 0 \le t \le 1 \\ t - 1 & \text{for } t > 1 \end{cases}$$

   (a) Sketch a graph of $f(t)$ and $f'(t)$. Is $f(t)$ piecewise continuous? Averaged?

   (b) Express $f(t)$ in terms of the unit step function $u(t)$, and compute $f'(t)$ algebraically. Do you get the same answer?

3. (a) Consider the operator $D^2 + 3D + 2I$. Compute the unit impulse response directly, and using Laplace transform.

   (b) How would you use this to solve $x'' + 3x' + 2x = \cos t$ with rest initial conditions? Set up the relevant convolution integral.

   (c) Find the Laplace transform of the solution to $x'' + 3x' + 2x = \cos t$, with initial conditions $x(0) = 0$, $x'(0) = 1$.

4. Let $f(t) = \cos(2t) + e^{-t}\sin(3t)$. Compute $F(s) = \mathcal{L}[f]$ and sketch the pole diagram.

5. Compute the inverse Laplace transforms of $F_1(s) = 1/(s^2(s+1))$, $F_2(s) = e^{-s}/s^2$.