Math 18.06, r3/5
Problems #3
March 8, 2013

1. Let U be a matrix in echelon form:

$$U = \begin{pmatrix} 1 & 2 & 0 & 4 & 5 \\ 0 & 0 & 1 & 7 & 8 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

   Describe a basis for each of the four fundamental subspaces. If A reduces to U under elimination, what can you say about the fundamental subspaces of A?

2. Suppose that A is a 3×3 matrix. What are the possible sets of dimensions for the four fundamental subspaces? For each possibility, give an example of such a matrix and describe geometrically the corresponding transformation.

3. Suppose you were handed a 3×5 matrix. What procedure would you use to compute a basis for each of the four fundamental subspaces?

4. Let A be the product

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}.$$

   Find the four fundamental subspaces first for the factor on the right, and then for A itself. How would your answer change if the last row of the right-hand factor were replaced with 0s?

5. Suppose A is a $m \times n$ matrix, and you know there is a vector $\mathbf{b}$ for which $A\mathbf{x} = \mathbf{b}$ has no solution. What does this tell you about the four fundamental subspaces?

6. Every rank 1 matrix can be written in the form $\mathbf{u}\mathbf{v}^T$ (column times row). Explain this for the rank-1 matrix I spotted at the store last weekend:

$$\left(\begin{array}{r|r|r|r} 3\text{ t} & 1\text{ T} & 1/16\text{ c} & 1/64\text{ qt} \\ 12\text{ t} & 4\text{ T} & 1/4\text{ c} & 1/16\text{ qt} \\ 24\text{ t} & 8\text{ T} & 1/2\text{ c} & 1/8\text{ qt} \\ 48\text{ t} & 16\text{ T} & 1\text{ c} & 1/4\text{ qt} \end{array} \right)$$