Math 18.06, r3/5
Problems #5
March 19, 2013

1. (a) A^TA is invertible if _____.
   (b) Say A is a square orthogonal matrix. Must A^T be as well? What if A is not square?
   (c) Suppose $A = LDU$. What are the determinants of each of the three factors on the right?
   (d) What about $A = QR$?
   (e) Give an example of a 4×4 matrix A for which $\det A$ is not just (product of downward diagonals) - (product of upward diagonals). Hint: you can use a permutation matrix.

2. Find a best fit parabola $y = ax^2 + bx + c$ through the four points $(-1, 0)$, $(0, 0)$, $(1, 0)$, $(2, 1)$ (or at least explain how you would). Could you find a best fit curve of the form $y = a + be^{cx}$ through these points?

3. Let A be the matrix
$$A = \begin{pmatrix} 1 & 0 \\ 2 & -4 \\ 3 & 2 \end{pmatrix}.$$
What could you add as the third column to get a matrix with all columns orthogonal to each other? What else do you have to do to get an orthogonal matrix?

4. Find an orthonormal basis for the subspace of $\mathbb{R}^3$ defined by $x + 2y + 3z = 0$. Once you have an answer, how can you check it?

5. Find the QR decomposition for $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$.

6. (a) Let $A = \begin{pmatrix} 0 & 1 & 2 \\ -1 & 0 & 3 \\ -2 & -3 & 0 \end{pmatrix}$. Check that $\det A = 0$.
   (b) Suppose A is any skew-symmetric matrix, i.e. $A^T = -A$. Show that if n is odd, then $\det A = 0$. What can you say if n is even?

7. (a) Let
$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 3 & 4 \end{pmatrix}.$$
   What is $\det A$?
   (b) Suppose that
$$M = \left(\begin{array}{c|c} A & B \\ \hline 0 & C \end{array}\right)$$
   is any 4×4 matrix composed of four 2×2 blocks. Show that $\det M = \det A \cdot \det C$. What happens if
$$M = \left(\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right)?$$